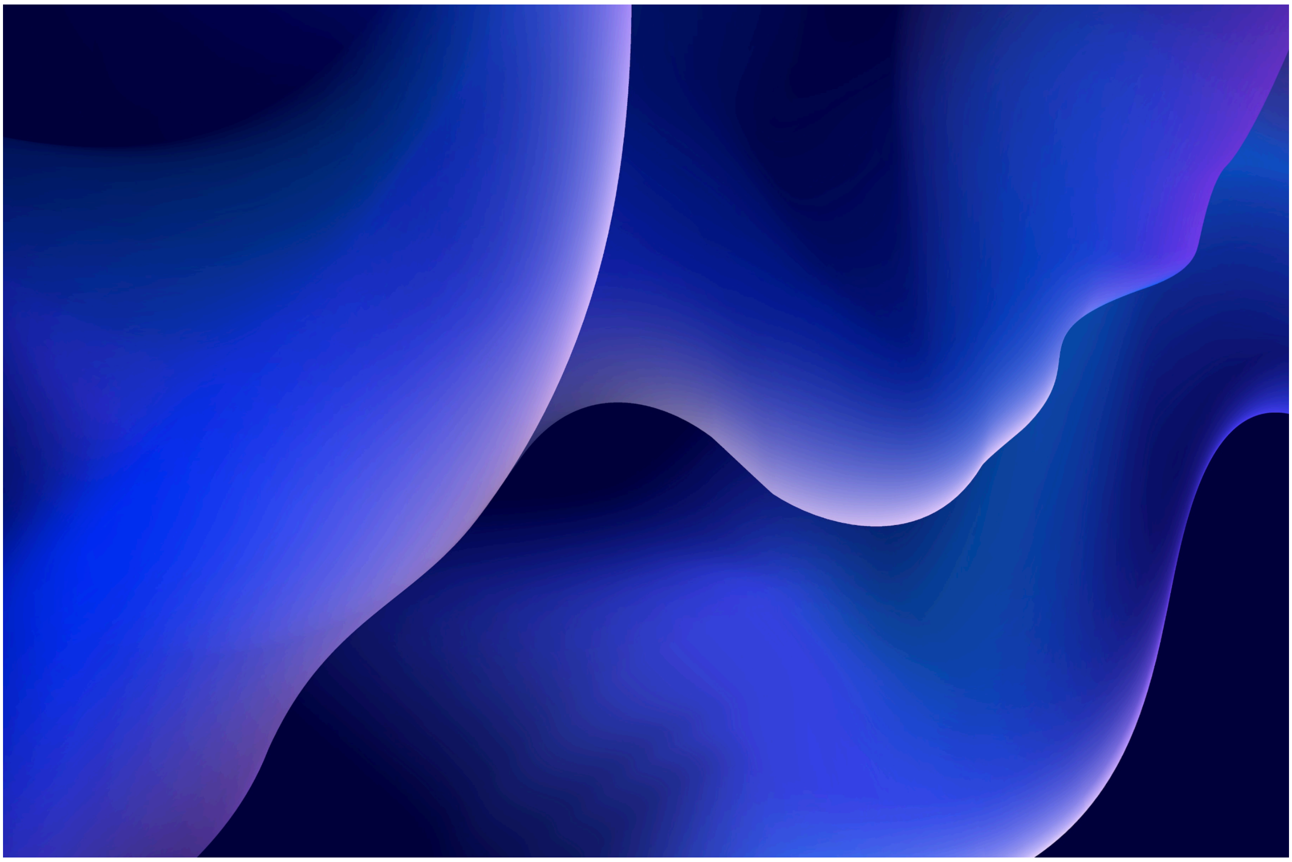
Electro Weak



The Electro Weak Standard Model

Content:

Chapter 1: Introduction to the week interaction

Outline:
- Basic Characteristic of the week interaction
- Reminders about parity violation in week interactions
- The V-A structure of the charged week currents
- Neutral Current, mixing, CP violation, neutrinos etc

Introduction to the week interaction

$$\Delta^{++} \to p\pi^{+} \longrightarrow \tau \sim 10^{-23}\ \mathrm{s},$$ .... Strong Interaction

$$J/\psi \to e^{+}e^{-} \longrightarrow \tau \sim 10^{-20}\ \mathrm{s},$$

$$\Sigma^{0} \to \Lambda\gamma \longrightarrow \tau \sim 10^{-20}\ \mathrm{s},$$ .... Electromagnetic Interaction

$$\pi^{0} \to \gamma\gamma \longrightarrow \tau \sim 10^{-16}\ \mathrm{s},$$

$$\tau^{-} \to \mu^{-}\bar{\nu}_{\mu}\nu_{\tau} \longrightarrow \tau \sim 10^{-13}\ \mathrm{s},$$

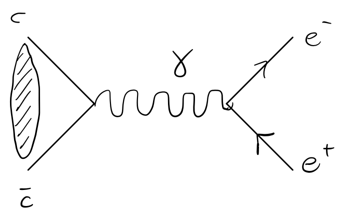
$$\Sigma^{-} \to n\pi^{-} \longrightarrow \tau \sim 10^{-10}\ \mathrm{s},$$ .... Week Interaction

$$\pi^{-} \to \mu^{-}\bar{\nu}_{\mu} \longrightarrow \tau \sim 10^{-8}\ \mathrm{s},$$

$$\mu^{-} \to e^{-}\bar{\nu}_{e}\nu_{\mu} \longrightarrow \tau \sim 10^{-6}\ \mathrm{s},$$

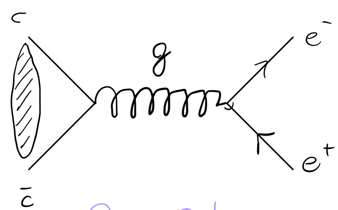
$$n \to pe^{-}\bar{\nu}_{e} \longrightarrow \tau \sim 10^{3}\ \mathrm{s}.$$

Let's focus on something



We know that the mediator is a PHOTON

Why it cannot be a GLUON?



Ozi Rule ⇒
Diagrams with disconnected quark lines are suppressed relative to connected ones

Let's consider the weak interaction

$$\frac{\Sigma(\Lambda \to p\pi)}{\Sigma(\Sigma \to n\pi)} \approx \frac{10^{-23}}{10^{-10}} = 10^{-13} = \left(\frac{\alpha_w}{\alpha_s}\right)$$

Why that specific decay we selected?

$$\to d\Gamma = \underbrace{\frac{1}{2E}\frac{1}{2S_x+1}}_{\text{SAME}} \sum \mathcal{M}\mathcal{M}^* \underbrace{d\Omega}_{\text{SAME}}$$

For the 2 decay
$m_p \simeq m_n$
$m_\pi \simeq m_\pi$

$\to$ the difference is the Physics

⇒ Coupling for the weak interaction : α_W~10^{-6}

Pay Attention: The weak coupling constant scale with the mass of the MEDIATOR

The mediator $\quad \Pi \propto \frac{g^{\mu\nu}}{\cancel{q^2} - M_W^2}$ $\quad q \ll \Rightarrow$ constant
$\quad q \gg \Rightarrow$ scale

Let's Focus on all the decay instead!

⇒ The observed differences in lifetime all related to the Phase Space of the decays

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Electro Weak interaction makes the Sun shining:

W Int $\quad pp \to D\, e^+ \nu_e \quad \to$ Drives the Sun's lifetime

S+E Int $\quad D + p \to {}^3He + \gamma$

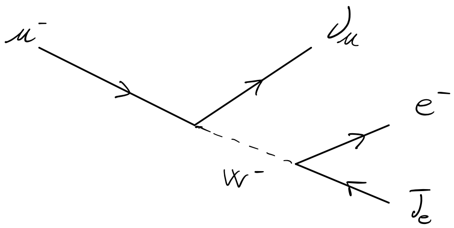
Fusion $\quad \begin{cases} {}^3He + {}^3He \to {}_2^4He + 2p \\ {}^3He + {}^4He \to {}^7Be + \gamma \end{cases}$ $\quad \Big\| \quad$ ${}^7Be + e^- \to {}^7Li + \nu_e$

Bigger Stars: $\quad {}^4He + {}^4He + {}^4He \to {}^{12}C$

Physics Model

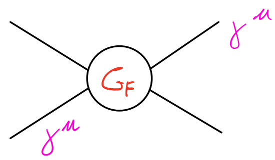
In the early 30's Fermi developed a model for the weak interaction.

Muon Decay:



We know it's like this nowadays

Fermi instead contracted the W^- in a point



So in analogy with electrodynamics:

$$\boxed{\mathcal{M}_{WEAK} = G_F \left(\bar{u}_n \gamma^\mu u_p\right)\left(\bar{u}_{\nu_e} \gamma_\mu u_e\right)}$$

$\rightsquigarrow$ We still miss $(1-\gamma^5)$ but Fermi didn't know Parity Violation

Decay widths dimensionally: must have an energy to power 5 (E^5)

We can build Γ, and by measuring τ_n or τ_μ, we can have the G_F measurement.

For example:

$$\Gamma(n \to p e^- \bar{\nu}_e) = \frac{G_n^5 E_0^5}{30\pi^3}$$

$$\Gamma(\mu^- \to e^- \bar{\nu}_e \nu_\mu) = \frac{G_\mu^2 m_\mu^5}{192\pi^3}$$

Experimental measurements yield the equality of the two coupling constants.

I. Basic Properties of the weak interaction

A decay of a muon proceeding through an e^- and a p has not been observed yet. $(\mu \to e^- p)$

Hence we define a leptonic number preserved by all interactions

Q	$L_e=1$	$L_\mu=1$	$L_\tau=1$
0	ν_e	ν_μ	ν_τ
-1	e^-	μ^-	τ

Similarly we never observed the baryon number to be modified in any process.

Hence a baryonic number defined in terms of quarks is:

$$\boxed{B = \frac{1}{3}(N_q - N_{\bar{q}})}$$

Also we searched for Lepton Flavor Violation

Theoretically we can calculate Γ for $\mu \to e\gamma$

$$\Gamma(\mu \to e\gamma) \approx \underbrace{\frac{G_F^2 m_\mu^5}{192\pi^3}}_{\mu-\text{decay}} \underbrace{\left(\frac{\alpha}{2\pi}\right)}_{\gamma-\text{vertex}} \underbrace{\sin^2 2\theta \sin^2\left(\frac{1.27\Delta m^2}{M_W^2}\right)}_{\nu-\text{oscillation}}$$

$$\approx \frac{G_F^2 m_\mu^5}{192\pi^3} \frac{3\alpha}{32\pi} \left(\frac{\Delta m_{23}^2 s_{13} c_{13} s_{23}}{M_W^2}\right)^2$$

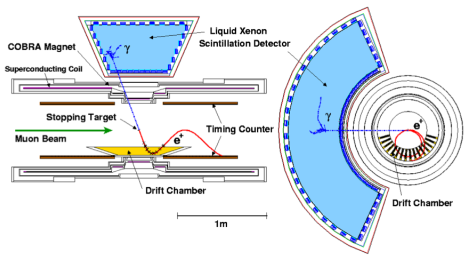
relative probability ~ 10^{-54}

Then experimentally

MEG Experiment

$$\frac{\Gamma(\mu^+ \to e^+\gamma)}{\Gamma(\mu^+ \to e^+\nu\bar{\nu})} \leq 2.4 \times 10^{-12}$$

• Stop muons and look at a gamma (Liquid scintillator) produced in correlation with an electron (magnetic spectrometer).



I.2 The parity: definition and some properties

Space Reflection Operator

$$P\,\psi(t,\vec{r}) = \psi'(t,r) = \psi(t,-r)$$

It's an observable?

$\rightsquigarrow$ We should find the heigenvalue

We suppose that there are heigenvalues $\quad P\,\psi(t,\vec{r}) = \eta_P\,\psi(t,\vec{r})$

Do it twice $\Rightarrow$

$$\left.\begin{aligned} PP\psi(t,\vec{r}) &= P\psi(t,-r) = \psi(t,r) \\ PP\psi(t,\vec{r}) &= P\eta_P\,\psi(t,r) = \eta_P\,P\psi(t,r) = \eta_P^2\,\psi(t,r) \end{aligned}\right\} \eta_P^2 = \pm 1$$

Vocabulary

What parity i revert space $\longrightarrow$

$\vec{r} \to -\vec{r}$

$t \to t$

$\vec{p} \to -\vec{p}$

$\vec{S} \to \vec{S}$

Vector quantities which are changing sign after parity TRANSFORM are called VECTORS

If unchanged, they are called PSEUDOVECTORS OR AXIAL VECTORS

Table

	$S=0$	$S=1$
$\eta_P=+1$	SCALAR σ, H	AXIAL
$\eta_P=-1$	PSEUDOSCALAR $\pi^+, \pi^-, \pi^0, K^\pm$	VECTOR e, K^*, γ

$\eta(\pi^{\pm}) = -1$

$\eta_P(p)$

We should define $|p\rangle = |uud\rangle$

So $\Rightarrow \quad \eta(p) = \eta_p(u) \cdot \eta_p(u) \cdot \eta_p^{orb}(d) \cdot \eta_p^{orb}(uu, d) \cdot \eta_p^{orb^2}(ud, u)$

$|p\rangle$ is the grand state :

as such $\quad \eta_p(qq^{(1)}, q) = (-1)^{\ell} = (-1)^0 = +1$

So $\eta_p(p) = (+1)^3 \cdot +1 = +1$

Calculate the parity eigenstates of 2 pions in interaction

$\eta_p(\pi^+\pi^-) = \eta_p(\pi^+)\,\eta_p(\pi^-) \cdot \eta_p^{orb}(\pi^+\pi^-)$

$= (-1) \cdot (-1) \cdot \eta_p^{orb}(\pi^+\pi^-) = (-1)^{\ell}$

We will use $\quad \ell=0$ s-wave
$\ell=1$ p-wave
$\ell=2$ d-wave
$\ell=3$ n-wave

So $\eta(\pi^+\pi^-\pi^+) = -\eta_p^{orb}(\pi^+\pi^-\pi^0) \quad \ell=0 \quad \eta_p(\pi^+\pi^-\pi^0) = -1$

The θ/τ puzzle

2 decays were observed in the 50's through weak interaction

$\tau \to \pi^+\pi^+\pi^- \qquad P = -1$

$\theta \to \pi^+\pi^0 \qquad P = +1$

TAU THETA PUZZLE

These particle has same m, s, τ

SAME PARTICLE $\Rightarrow$ Weak Interaction violates Parity

Wu experiment

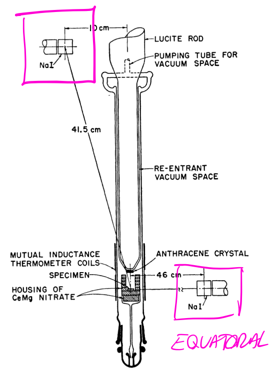
$\boxed{1956}$ Testing for this breaking symmetry

Experiment

- ^{60}Co nuclei were polarized $\Rightarrow$ Spins aligned in a particular direction

- ^{60}Co undergo beta decay $\quad ^{60}Co \to {}^{60}Ni^{**} + e^- + \bar{\nu}_e$
  $\to {}^{60}Ni + e^- + \bar{\nu}_e + \gamma\gamma$

POLAR



EQUATORIAL

Fig. 1. Schematic drawing of the lower part of the cryostat.

- Gamma Rays: 1. Electro Magnetic was known to respect parity conservation
  $\to \gamma$ would be emitted equally in all directions
  $\to$ Comparing distribution of γ with e^- distrib

  2. Not-isotropically distributed? ^{60}Co spins well-alignment measurment

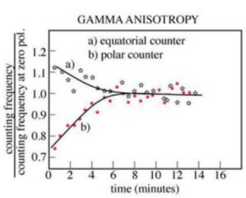
- Measurements: The experiment counted the rate of emission for gamma rays and electrons in 2 distinct directions and compare their values

  $\to$ The measurement has been done over time and with the polarizing field oriented in 2 opposite directions.
  $\to$ At $t=0$ switch off magnetic field

  $\to$ At $t=0$ we expect the γ to be measured MORE in the EQUATORIAL with RESPECT TO POLAR (polarization is working)

- Angular momentum conservation: e^- should be emitted symmetrically in all directions
  $\to$ IF NOT (but γ yes) PARITY VIOLATION

- Measurements

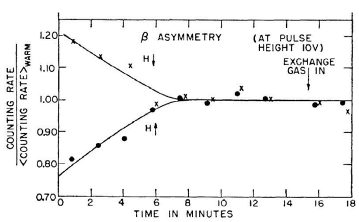


at $t=0$ $\#\gamma$ a) $\gg \#\gamma$ b)

SPIN POLARIZ IS WORKING

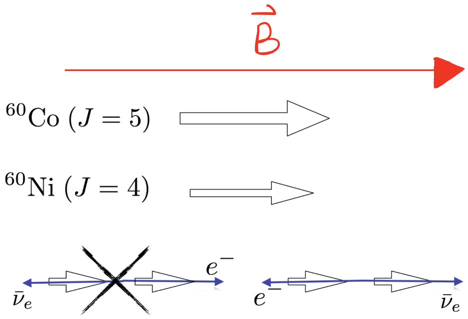
Then for two check B

Just POLAR counter



$\#\beta$ polar counter $B\downarrow \gg \#\beta$ polar counter $B\uparrow$
e^- prefer to be emitted opposite to B

- <u>Explanation</u>:



The spins are aligned with B.

$\Rightarrow$ For conservation of momentum 2 POSSIBLE configurations

$\Rightarrow$ Since e^- prefer to be emitted opposite to B, delete one possibility!

Goldhaber experiment

^{152}Eu decays for electron capture : $^{152}Eu + e^- \to {}^{152}Sm^* + \nu_e \to {}^{152}Sm + \gamma + \nu_e$

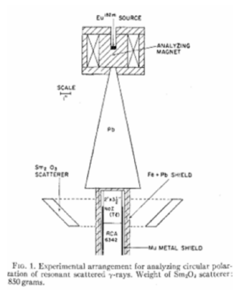
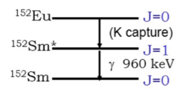


Fig. 1. Experimental arrangement for analyzing circular polarization of resonant scattered γ-rays. Weight of Sm2O3 scatterer: 830 grams.

$^{152}Eu(J=0) + e^- \to {}^{152}Sm^*(J=1) + \nu$ (K capture)
$\quad \hookrightarrow {}^{152}Sm(J=0) + \gamma$



The spins of all final states particles are constrained. The gammas aligned with the ^{152}Sm are selected and their polarization is measured.

- Spins constrains: the spin of the initial electron defines the initial and final states.
  (For the experiment we will use $\vec{L}_e = 0$, e^- it's not a system of 2, so $L_e = 0$)

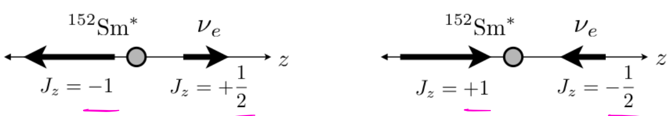
$\to \vec{L}_e = 0$ and also $\vec{L}_{\nu_e} = 0$. Also we will measure all the Sm with $\vec{L}_{Sm} = 0$.

$$\vec{J}_{Eu} + \vec{S}_e = \vec{J}_{Sm} + \vec{J}_{\nu_e}$$

$$\vec{0} + \vec{L}_e + \vec{S}_e = \vec{L}_{Sm} + \vec{S}_{Sm} + \vec{L}_{\nu_e} + \vec{S}_{\nu_e}$$

$$\vec{0} + \vec{\tfrac{1}{2}} = \vec{0} + \underline{\underline{\vec{1}}} + \vec{0} + \underline{\underline{\vec{\tfrac{1}{2}}}} \Rightarrow 2 \text{ conf}$$

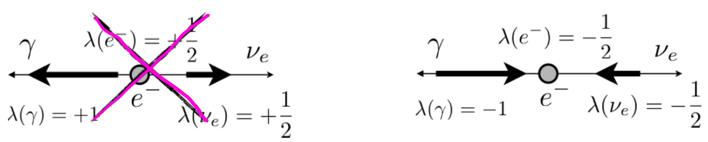
There are just only 2 config as possible



The K-capture is followed by the excited Samarium decay

$$^{152}Sm^*(J=1) \to {}^{152}Sm(J=0) + \gamma$$

The γ (massless vector boson) has 2 possibles polarizations, which manifest in the 2 and ONLY 2 possible configurations of helicities

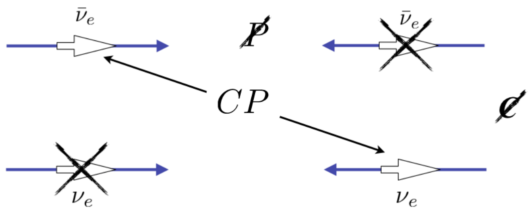


Result

From the γ polarization measurement, Goldhaber et al show that only left-handed neutrinos are found in β decays

Summary

- Parity is violated in weak interaction
- One gets from all experimental results the following picture



Any theory of weak interaction shall include these properties

The V-A structure of the weak force

In general only 5 combination of 2 spinors and γ-matrices complies with LORENTZ INVARIANCE

Type	Expression	Components	Mediating Boson
Scalar	$\bar{\Psi}\Phi$	1	Spin 0
PseudoScalar	$\bar{\Psi}\gamma^5\Phi$	1	Spin 0
Vector	$\bar{\Psi}\gamma^\mu\Phi$	4	Spin 1
Axial Vector	$\bar{\Psi}\gamma^\mu\gamma^5\Phi$	4	Spin 1
Tensor	$\bar{\Psi}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)\Phi$	6	Spin 2

Scalar: Higgs Field

Pseudo Scalar: Strong Force with π as mediator

Vector: All QCD

Axial: Weak

Tensor: Gravitational

Testing vertex structures compliant with P violation

Vector Interaction

$$\bar{\Psi}\gamma^\mu\psi = \bar{\Psi}\underbrace{(P_L+P_R)}_{=1}\gamma^\mu\underbrace{(P_L+P_R)}_{=1}\psi \overset{!}{=} \bar{\Psi}_R\gamma^\mu\Psi_R + \bar{\Psi}_L\gamma^\mu\Psi_L$$

- $P_L = \frac{1}{2}(1-\gamma^5)$
- $P_R = \frac{1}{2}(1+\gamma^5)$

⤳ No selection of chirality states. Electromagnetism case.

Vector - Axial Vector interaction

$$\bar{\Psi}(1-\gamma^5)\psi = \bar{\Psi}\underbrace{(P_L+P_R)}_{=1}\gamma^\mu(1-\gamma^5)\underbrace{(P_L+P_R)}_{=1}\psi$$

$$\overset{!}{=} 2\bar{\Psi}(P_L+P_R)\gamma^\mu(P_L^2+P_LP_R)\psi$$

$$\overset{!}{=} 2\bar{\Psi}_L\gamma^\mu\Psi_L$$

$1-\gamma^5 = 2P_L$

⤳ Selection of chirality states. Only LL allowed for particles ≫ Natural Candidate for Weak Inter.

Parentesis: Helicity v.s. Chirality

Helicity is easy, the projection of spin on momentum, but if you go fast enough you will change the momentum, so it's not a good thing

Let's try to relate H with Chirality.

Let's start with solution $E>0$ of Dirac equation (momentum along z)

$$u_1 = \sqrt{E+m}\begin{pmatrix}1\\0\\\frac{p}{E+m}\\0\end{pmatrix} \qquad u_2 = \sqrt{E+m}\begin{pmatrix}0\\1\\0\\-\frac{p}{E+m}\end{pmatrix}$$

SPIN UP   SPIN DOWN

Define HELICITY:

$$\hat{h} = \frac{1}{2}\vec{p}\cdot\vec{\sigma} = \frac{1}{2}p\begin{pmatrix}\sigma_3 & 0\\0 & \sigma_3\end{pmatrix} \quad \text{with} \quad \sigma_3 = \begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}$$

So $\hat{h} = \frac{p}{2}\begin{pmatrix} 1 & & & \\ & -1 & & \\ & & 1 & \\ & & & -1 \end{pmatrix}$ and so

- $\hat{h}\, u_1 = \frac{1}{2} u_1$
- $\hat{h}\, u_2 = -\frac{1}{2} u_2$

u_1, u_2 are helicity eigenstates

Let's project those states with the chirality projectors P_L, P_R

$$P_L = \frac{1}{2}(1-\gamma^5) \rightarrow P_L = \frac{1}{2}\begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{pmatrix} \rightarrow P_L u_1 = \frac{1}{2}\sqrt{E+m}\begin{pmatrix} 1-\frac{p}{E+m} \\ 0 \\ -1+\frac{p}{E+m} \\ 0 \end{pmatrix}$$

$$P_R = \frac{1}{2}(1+\gamma^5) \rightarrow P_R = \frac{1}{2}\begin{pmatrix} 1 & 0 & +1 & 0 \\ 0 & 1 & 0 & +1 \\ +1 & 0 & 1 & 0 \\ 0 & +1 & 0 & 1 \end{pmatrix} \rightarrow P_R u_1 = \frac{1}{2}\sqrt{E+m}\begin{pmatrix} 1+\frac{p}{E+m} \\ 0 \\ +1+\frac{p}{E+m} \\ 0 \end{pmatrix}$$

Hence $u_1 = P_L u_1 + P_R u_1 = \frac{1}{2}\left(1-\frac{p}{E+m}\right)\sqrt{E+m}\begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix} + \frac{1}{2}\left(1+\frac{p}{E+m}\right)\sqrt{E+m}\begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}$

Remind that u_1 is spin up

$\vec{s}$

$\vec{p}_z$

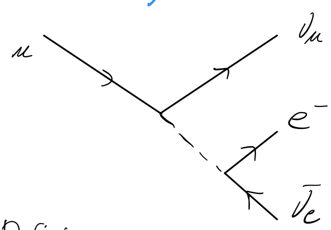
$$\boxed{\frac{1}{2}\left(1-\frac{p}{E+m}\right)\sqrt{E+m}\; u_L + \frac{1}{2}\left(1+\frac{p}{E+m}\right)\sqrt{E+m}\; u_R}$$

Helicity is a linear combination of chirality eigenstates

1. For massless particles helicity is chirality $\rightarrow u_L = 0$
2. For ULTRA RELATIVISTIC particles helicity is chirality
3. The heavier is a particle, the larger is the mixing of chiral states for a given helicity

$m \rightarrow \infty$ the $\frac{p}{E+m} \rightarrow 0$ and u_L and u_R FACTORS become similar.

Muon Decay



$$d\Omega = \prod_{i=1}^{3} \frac{d\vec{p}_i}{(2\pi)^3 2E_i} \cdot (2\pi)^4 \cdot \delta^{(4)}\left[\vec{P} - \sum_i \vec{p}_i\right]$$

Defining:

$$p_\mu = P$$

$$p_e, p_{\bar{\nu}_e}, p_{\nu_\mu} = p_1, p_2, p_3$$

$$|\mathcal{M}|^2 = \frac{1}{S_\mu + 1} \cdot \sum_{spins} \mathcal{M} \cdot \mathcal{M}^*$$

Matrix element of the transition

$$\mathcal{M} = \frac{G}{\sqrt{2}} \cdot \left[u(\bar{p}_3)\gamma^\mu(1-\gamma^5)u(P)\right] \cdot \left[u(\bar{p}_2)\gamma_\mu(1-\gamma^5)v(p_2)\right]$$

Summing over the initial and final spins, we are ending with

$$|\bar{\mathcal{M}}|^2 = \frac{G^2}{4} \mathcal{M}^{\mu\nu} \mathcal{N}_{\mu\nu}$$

$$\mathcal{M}^{\mu\nu} = \mathrm{Tr}\{\not{p}_3 \gamma^\mu (1-\gamma^5)(\not{P} + m_\mu)\gamma^\nu(1-\gamma^5)\}$$

By explicit computation

$$d\Gamma = \frac{1}{2E}|\bar{\mathcal{M}}|^2 dQ = \frac{64\,G^2}{2E} \cdot (P\,p_2)(p_3\,p_1)\,dQ$$

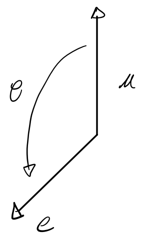
Hence after integration

$$\Gamma(\mu^- \to e^- \bar{\nu}_e \nu_\mu) = \frac{G_\mu^2 m_\mu^5}{192\pi^3}$$

Twist Experiment

Has measured several billions of muon decay.

→ They measured the electron energy, other than the muon energy and the angular distrib relative to the muon spin momentum



Michel parameter $\rho = \underline{\underline{0.7508}}$

$\rho = \underline{\underline{0.75}}$ V − A prediction

Mediating the Force

In QFT's view of interactions, they shall be mediated throw a vector boson.
We shall modify slightly the matrix element

$$\mathcal{M} = [\frac{g}{\sqrt{2}}\bar{u}(p_3)\frac{1}{2}\gamma^{\mu}(1-\gamma^5)u(p)].(\frac{1}{M_W^2 - q^2}).[\frac{g}{\sqrt{2}}\bar{u}(p_1)\frac{1}{2}\gamma_{\mu}(1-\gamma^5)v(p_2)] \ ,$$

· q^2 is the energy exchanged in the interaction

$$\Longrightarrow \quad G_F = \frac{\sqrt{2}\, g^2}{8 M_W^2}$$

Considerations:

- $q^2 \ll M_W^2$:
  - G_F is the coupling constant
  - The propagator disappears
- V-A works for low energies
- Taking $g = e$ we will have $M_W \sim \cancel{40 \text{ GeV}}$

  In fact $e = g\sin(\theta_W) \to M_W \sim 80$ GeV

THE WEAK INTERACTION IS NOT WEAK because of $g^2 \ll e^2$ BUT BECAUSE OF M_W

Recall the pion decay

What other processes similar to $\mu^- \to e^- \bar{\nu}_e \nu_\mu$ we can imagine with the lepton?
We had

$$\Gamma(\pi^+ \to \ell^+ \nu_\ell) = \frac{G_F^2}{8\pi} f_\pi^2 m_\ell^2 m_\pi \left[1 - \frac{m_\ell^2}{m_\pi^2}\right]^2$$

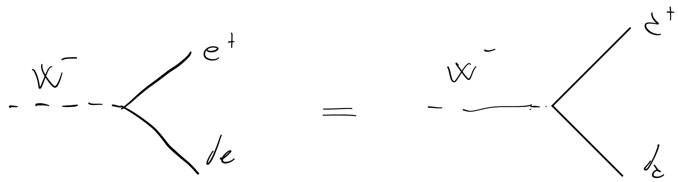
let's try to compare $\pi \to$ electron and $\pi \to$ muon

$$\frac{\Gamma(\pi^+ \to e^+ \nu_e)}{\Gamma(\pi^+ \to \mu^+ \nu_\mu)} = \left(\frac{m_e}{m_\mu}\right)^2 \left[\frac{m_\pi^2 - m_e^2}{m_\pi^2 - m_\mu^2}\right]^2 = 1.26 \times 10^{-4}$$

Expectation $\Rightarrow$ Should be much more probable the electron

The experimental observation: $B(\pi^+ \to e^+ \nu_e) = 1.23 \cdot 10^{-4}$

If we look at the amplitude of those processes we can see that:



They are the same

BUT STILL $\Rightarrow$ For e^- is much more difficult to be in the WRONG HELICITY

Intermediate conclusions

- The weak interaction (charged currents) maximally violates parity
- Correct shape of weak interaction is V-A
- Weak interaction selects LH particle and RH antiparticle
- In the UR limit, chirality = helicity
- Only LEFT HANDED NEUTRINOS EXIST

What the model MUST include as well?

CABIBBO MODEL

The lifetime of STRANGE PARTICLES can be accounted for if we were to consider a mixing between s and d quarks, governed by the mixing parameter θ_c.
We say that the energy eigenstates are a linear combination of the weak eigenstates

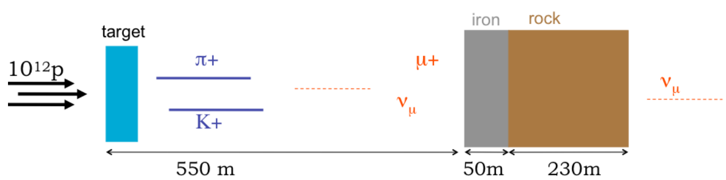
$$\begin{pmatrix} u \\ d_c \end{pmatrix} = \begin{pmatrix} u \\ d\cos\theta_c + s\sin\theta_c \end{pmatrix}$$

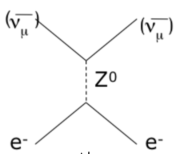
$\rightarrow$ The couplings are modified such that

$$ud : \quad G_F \rightarrow G_F\cos(\theta_c) \qquad \sin(\theta_c) \sim 0.22 \quad \text{Measured}$$
$$us : \quad G_F \rightarrow G_F\sin(\theta_c)$$

NEUTRAL CURRENTS

GARGAMELLE:
discovered interactions of ν_μ without charged muon in the final state



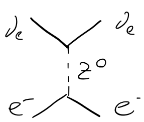


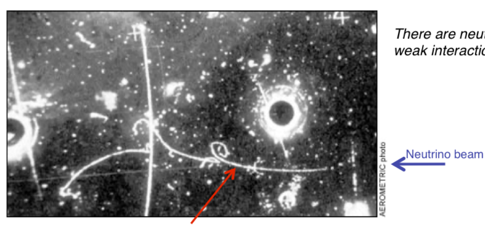
With electronic neutrinos, charged and neutral currents are competing.

With muonic neutrinos, only neutral currents. This is an unambiguous signature. $\rightarrow$ of course if you DO NOT MEASURE MUONS it's TRUE

$\Downarrow$

it's weird because we knew already





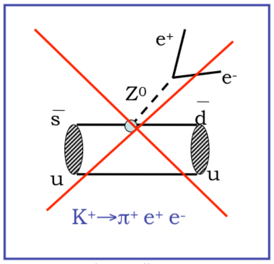
There are neutral currents in the weak interaction

electron detected from a beam of JUST MUONIC NEUTRINOS

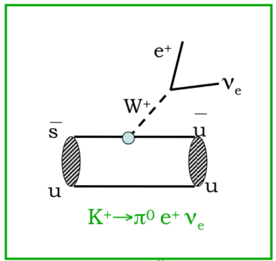
400 MeV e- aligned with incoming ν beam

GIM MECHANISM

Neutral currents do exist but they do NOT occur through 3-level Flavour CHANGING process:



sd coupling



su coupling

Whit the GIM mechanism we can solve the problem

$$\psi\bar{\psi} = (u \quad d_c)\begin{pmatrix}\bar{u}\\ \bar{d}_c\end{pmatrix} = u\bar{u} + dd\cos^2\theta_C + s\bar{s}\sin^2\theta_C + (sd + \bar{s}d)\sin\theta_C\cos\theta_C$$

Let's suppose now that we have a 4th quark in the game and rank it in a second doublet together with the strange quark

$$\varphi = \begin{pmatrix}c\\ s_c\end{pmatrix} = \begin{pmatrix}c\\ s\cos\theta_c - d\sin\theta_c\end{pmatrix}$$

$$\varphi\bar{\varphi} = c\bar{c} + d\bar{d}\sin^2\theta_c + s\bar{s}\cos\theta_c - (s\bar{d} + d\bar{s})\sin\theta_c\cos\theta_c$$

The neutral couplings are eventually written as:

$$\bar{\Phi}\Phi + \bar{\Psi}\Psi = c\bar{c} + s\bar{s} + d\bar{d} + u\bar{u}$$

No FCNC (Flavour Changing Neutral Current)

The charged currents have to be written with the proper mixing:

$$\begin{pmatrix}d'\\ s'\end{pmatrix}_{EW} = \begin{pmatrix}\cos\theta_C & \sin\theta_C\\ -\sin\theta_C & \cos\theta_C\end{pmatrix}\begin{pmatrix}d\\ s\end{pmatrix}_{MASS}$$

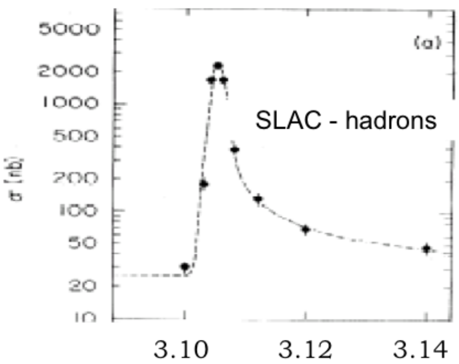
As a by-product there shall exist a fourth quark: the charm quark

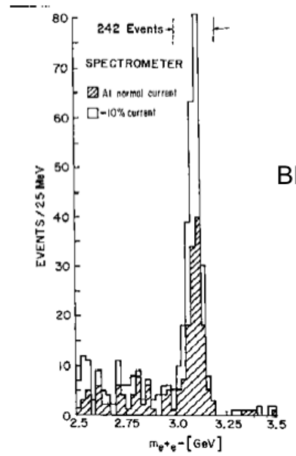
HEAVY FLAVOURS

In 1974 at SLAC and Brookhaven narrow resonance $J/\psi = |c\bar{c}\rangle$

$m \sim 3.1$ GeV

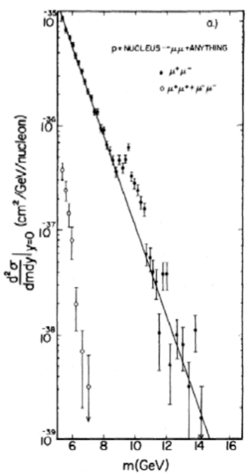
$\tau \sim 10^{-20}$ sec



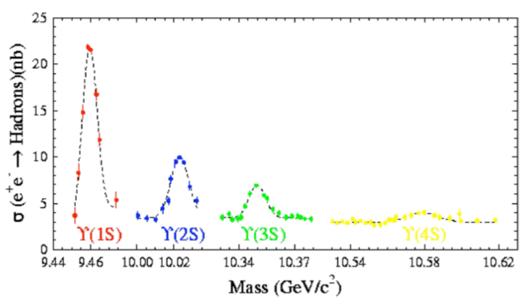


BNL - muons

The early discovery series of 9.5 ~ 10.5 GeV $b\bar{b}$ resonances



B-factories : the Υ(4S) decays into a $B^0\bar{B}^0$ pair or a B^+B^- pair (CLEO, BaBar, BELLE)



ElectroWeak Physics 49

Then followed

- The tau leptonic neutrino
- Z boson

CP VIOLATION

A K^0 can be produced in strong interaction processes:

$$\pi^- p \to K^0 \Lambda$$

$$\pi^+ p \to \bar{K}^0 K^+ p$$

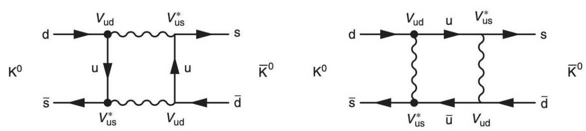
K^0 and it's antiparticle are different particles

Neutral mesons mixing and CP violation

$$K^0 \to \pi\pi \quad \text{with} \quad \tau_{\pi\pi} \approx 10^{-11}$$

$$K^0 \to \pi\pi\pi \quad \text{with} \quad \tau_{\pi\pi\pi} \approx 10^{-8}$$

In Quantum Mechanics K^0 and $\bar{K}^0$ can MIX



Two box diagrams for $K^0 \leftrightarrow \bar{K}^0$ mixing. There are corresponding diagrams involving all nine combinations of virtual up, charm and top quarks.

Hence the decaying particle is a mixture of the mass eigenstates:

$$K_2 = \frac{1}{\sqrt{2}} \left(|\bar{K}_0\rangle + |K_0\rangle \right)$$

$$K_2 = \frac{1}{\sqrt{2}} \left(|K_0\rangle - |\bar{K}_0\rangle \right)$$

Let's write the CP states

$$\begin{aligned} CP|K_1\rangle &= \frac{1}{\sqrt{2}}(CP|K^0\rangle + CP|\bar{K}^0\rangle) = \frac{1}{\sqrt{2}}(|K^0\rangle + |\bar{K}^0\rangle) = +|K_1\rangle \qquad CP = +1 \quad (\pi\pi) \\ CP|K_2\rangle &= \frac{1}{\sqrt{2}}(CP|K^0\rangle - CP|\bar{K}^0\rangle) = \frac{1}{\sqrt{2}}(|\bar{K}^0\rangle - |K^0\rangle) = -|K_2\rangle \qquad CP = -1 \quad (\pi\pi\pi) \end{aligned}$$

We will identify

$K_L \to \pi\pi$ as K^0_S K short

$K_2 \to \pi\pi\pi$ as K^0_L K long

The lifetime difference in K^0 system is simply related to the dynamical accident 2-body vs 3-body

$\Rightarrow$ MASS DIFFERENCE is the other consequence of mixing.

Can be seen in oscillations of neutral B.

Side Notes
Calculation

For Asymmetry Measurements

$\frac{A_S - A_L}{A_S + A_L}$ or something like this

In questa ipotesi di valori di p,q, in buon accordo con i dati sperimentali, si conclude essere:

$$\begin{aligned} |\tilde{K}_S\rangle &= \frac{|K^0\rangle + |\bar{K^0}\rangle}{\sqrt{2}} & |K^0\rangle &= \frac{|\tilde{K}_S\rangle + |\tilde{K}_L\rangle}{\sqrt{2}} \\ |\tilde{K}_L\rangle &= \frac{|K^0\rangle - |\bar{K^0}\rangle}{\sqrt{2}} & |\bar{K^0}\rangle &= \frac{|\tilde{K}_S\rangle - |\tilde{K}_L\rangle}{\sqrt{2}} \end{aligned} \tag{8.107}$$

Consideriamo ora una particella K_0 in stato iniziale e valutiamo la sua evoluzione temporale. Si ha:

$$\begin{aligned} |\psi(0)\rangle &= |K^0\rangle = \frac{|\tilde{K}_S\rangle + |\tilde{K}_L\rangle}{\sqrt{2}} \implies |\psi(t)\rangle = \exp(-i\mathrm{H}t)\,|\psi(0)\rangle = \\ &= \frac{1}{\sqrt{2}}\left[\exp(-i\mathrm{H}t)\,|\tilde{K}_S\rangle + \exp(-i\mathrm{H}t)\,|\tilde{K}_L\rangle\right] = \\ &= \frac{1}{\sqrt{2}}\left[\exp(-i(m_S - i\Gamma_S/2)t)\,|\tilde{K}_S\rangle + \exp(-i(m_L - i\Gamma_L/2)t)\,|\tilde{K}_L\rangle\right] = \\ &= \frac{1}{\sqrt{2}}e^{-im_S t}\left[e^{-\frac{\Gamma_S}{2}t}\,|\tilde{K}_S\rangle + e^{-i\Delta m t}e^{-\frac{\Gamma_L}{2}t}\,|\tilde{K}_L\rangle\right] \quad \text{con} \quad \Delta m = m_L - m_S \end{aligned} \tag{8.108}$$

Ciò permette di calcolare le ampiezze di probabilità relative alla possibilità di trovare al tempo $t > 0$ uno stato K^0, $\bar{K^0}$, K_S o K_L osservando la particella in questione. Per quanto riguarda l'ampiezza di probabilità di osservare un K_S si ha:

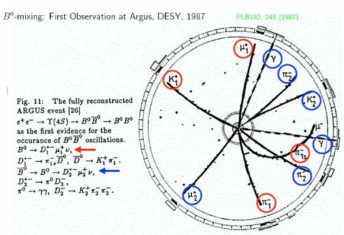
$A_S \Rightarrow$

$$\begin{aligned} A_S &\equiv A\left(K^0(t=0) \to K_S(t)\right) = \langle K_S|\psi(t)\rangle = \\ &= \langle \tilde{K}_S|\left(\frac{1}{\sqrt{2}}e^{-im_S t}\left[e^{-\frac{\Gamma_S}{2}t}\,|\tilde{K}_S\rangle + e^{-i\Delta m t}e^{-\frac{\Gamma_L}{2}t}\,|\tilde{K}_L\rangle\right]\right) = \\ &= \frac{1}{\sqrt{2}}e^{-im_S t}e^{-\frac{\Gamma_S}{2}t}\,\langle K_S|K_S\rangle + 0 = \frac{1}{\sqrt{2}}e^{-im_S t}e^{-\frac{\Gamma_S}{2}t} \end{aligned}$$

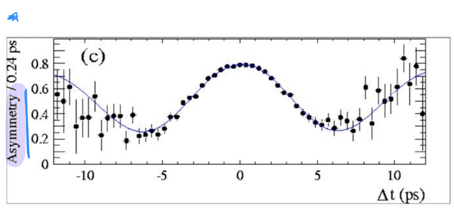
Dove si è utilizzato il fatto che $\langle K_i|K_j\rangle = \delta_{i,j}$ con $i,j = S,L$ in quanto $[|\tilde{K}_S\rangle, |\tilde{K}_L\rangle]$ costituiscono una base ortonormale.

Example B^0_d mesons:

Argus experiment (1987). First top quark evidence. Think about.

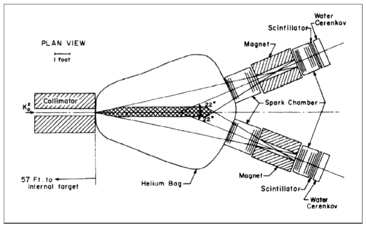


B factories (2000-). What a beautiful measurement.



Measurement of CP violation

→ What if we would measure K^0_L decays into two pions? That would be the indication that CP symmetry is violated in weak interaction

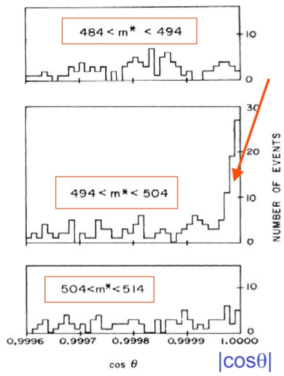


1964. Cronin et al. Far after the target, only K long survive. They measured:

$$|\eta_{+-}| = \frac{A(K^0_L \to \pi\pi)}{A(K^0_S \to \pi\pi)}$$

The experiment

The Measurement



Two body decay : in the K^0 center of mass system the two π are back to back : $|\cos\theta|=1$

$$|\eta_{+-}| = \frac{A(K^0_L \to \pi\pi)}{A(K^0_S \to \pi\pi)} = (2.271 \pm 0.017) 10^{-3}.$$

CP is violated. Slightly, but theory has to handle that feature of weak interaction. We do have a candidate now for matter-antimatter asymmetry in our universe.

Conclusions:

- The weak interaction (charged currents) maximally violates parity.
- A correct shape of weak interaction charged currents is *V-A* : the weak current can be written as $\bar{u}(p_1)\gamma^\mu(1-\gamma^5)u(p_2)$
- The weak interaction selects: Left handed particles and Right handed anti-particles.
- chirality/helicity: in the ultra relativistic case (*E>>m*) : chirality = helicity
- Only left handed neutrinos (and right handed anti-neutrinos) exist (if their mass is zero…)
- Charged currents can be mediated through a heavy *W* boson.
- The theory must find back these results.

✓ There are three families of quarks and leptons (12 matter particles).

✓ The propagators of the weak interaction are the intermediate *Z* and *W* bosons, massive.

✓ Quarks are mixing. The Model shall include a mass mixing matrix à la Cabibbo, generalized to 3 quark families.

✓ There is a mass hierarchy.

✓ *C* and *P* are maximally violated in weak interaction.

✓ There is also *CP* violation.

Chapter III - ElectroWeak Unification

Charged current V-A description of the weak interaction is successfull.

We might want to include the W propagator in the game.

There will be problems:

$$\sigma(\nu_\mu e \to \nu_\mu e) = \frac{G_F^2 s}{\pi} \to \infty \quad \text{when } s \to \infty \quad (q^2 \gg M_W^2)$$

QED works

The EM interaction is described under the photon exchange.

→ We can compute higher order Feynmann diagrams because of renormalization

→ Divergences are suppressed by a redefinition of the charge and mass of particles,

Property of local gauge invariance

Let's follow this example

A FREE elementary fermion is described with

$$\mathcal{L} = i\bar{\psi}(x)\gamma_\mu\partial^\mu\psi(x) - m\bar{\psi}(x)\psi(x)$$

GAUGE INVARIANCE = The symmetry is dictating the shape of the interaction

Let's hunt for those symmetries

Global Gauge Invariance

Let's consider $U(1)$ transformation

$$\psi(x) \to e^{i\Lambda}\psi(x)$$
$$\bar{\psi}(x) \to e^{-i\Lambda}\bar{\psi}(x)$$

The lagrangian density of the FREE fermion is invariant under this transformation:

$$\mathcal{L} = i\bar{\psi}(x)\gamma_\mu\partial^\mu\psi(x) - m\bar{\psi}(x)\psi(x) \longrightarrow i e^{-i\Lambda}\bar{\psi}(x)\gamma_\mu\partial^\mu e^{i\Lambda}\psi(x) - m e^{-i\Lambda}\bar{\psi}(x)e^{i\Lambda}\psi(x)$$

Since it's true $\Rightarrow$ Charge it's conserved (QED case it's electric)

$\longrightarrow$ Not POWERFUL ENOUGH FOR THE PURPOSE OF DESCRIBING INTERACTION

Comment:

The invariance under global continuous space-time transformation yelds to conserved quantities

$$U(\Lambda_1+\Lambda_2) = U(\Lambda_1)U(\Lambda_2)$$

$$UU^\dagger = \mathbb{1} \qquad U(\Lambda) = e^{-i\Lambda G} \qquad \text{with } G = G^\dagger$$

Example: Time

$$U(t) = e^{-it\frac{H}{\hbar}} \longrightarrow \text{Yelds to Energy Conservation}$$

Local Gauge Invariance

WE WILL MAKE $U(1)$ TRANSFORMATION LOCAL.

Let's chose the phase Λ as DEPENDENT of the coordinates $U(\Lambda(x))$

$$U_\lambda(1): \quad \psi(x) \longrightarrow \psi'(x) = e^{i\Lambda(x)}\psi(x)$$

$$\bar{\psi}(x) \longrightarrow \bar{\psi}'(x) = e^{-i\Lambda(x)}\bar{\psi}(x)$$

The lagrangian density of the FREE FERMION IS NO MORE INVARIANT under $U(1)$

$$\mathcal{L}^0 \xrightarrow{U_\lambda(1)} \mathcal{L}' = i e^{-i\Lambda(x)}\bar{\psi}(x)\gamma_\mu\partial^\mu\left(e^{i\Lambda(x)}\psi(x)\right) - m\cancel{e^{-i\Lambda(x)}}\bar{\psi}(x)\cancel{e^{i\Lambda(x)}}\psi(x) =$$

$$= i e^{-i\Lambda(x)}\bar{\psi}(x)\gamma_\mu\left(\partial^\mu e^{i\Lambda(x)}\right)\psi(x) + i e^{-i\Lambda(x)}\bar{\psi}(x)\gamma_\mu e^{i\Lambda(x)}\partial^\mu\psi(x) - m\cancel{e^{-i\Lambda(x)}}\bar{\psi}(x)\cancel{e^{i\Lambda(x)}}\psi(x)$$

$$= i e^{-i\Lambda(x)}\bar{\psi}(x)\gamma_\mu\, i\left(\partial^\mu\Lambda(x)\right)e^{i\Lambda(x)}\psi(x) + i e^{-i\Lambda(x)}\bar{\psi}(x)\gamma_\mu e^{i\Lambda(x)}\partial^\mu\psi(x) - m\cancel{e^{-i\Lambda(x)}}\bar{\psi}(x)\cancel{e^{i\Lambda(x)}}\psi(x)$$

$$\longrightarrow = i\bar{\psi}(x)\gamma_\mu\left(\partial^\mu + i\partial^\mu\Lambda(x)\right)\psi(x) - m\bar{\psi}(x)\psi(x)$$

Local gauge dogma:

In order to have $\mathcal{L}^0 = \mathcal{L}'$ we MUST ADD A new term to $\mathcal{L}^0$ NOT in the mass part (you will end up with the m multiplication)

$$\partial_\mu \to \partial_\mu - iX \quad \text{with} \quad X \xrightarrow{U(1)} X'$$

So

$$\mathcal{L}^0 + \mathcal{L}^1 = i\bar{\Psi}\gamma^\mu(\partial_\mu - iX)\Psi + m\bar{\Psi}\Psi$$

$$\xrightarrow{\qquad\qquad} U(1)$$

$$\to = i\bar{\Psi}\gamma^\mu(i\partial_\mu\Lambda)\Psi + \bar{\Psi}\gamma^\mu\partial_\mu\Psi - m\Psi\bar{\Psi} - i\bar{\Psi}\gamma^\mu_{(i}X'\Psi$$

$$\stackrel{!}{=} i\bar{\Psi}\gamma^\mu(i\partial_\mu\Lambda + \partial_\mu - iX')\Psi - m\bar{\Psi}\Psi$$

So in order to have GAUGE INVARIANCE

$$\partial_\mu - iX = \partial_\mu + i(\partial_\mu\Lambda - X')$$

$$\Rightarrow X = -(\partial_\mu\Lambda - X') \to X' = X + \partial_\mu\Lambda$$

$$\Rightarrow X \xrightarrow{U(1)} X' = X + \partial_\mu + \partial_\mu A$$

Let's note $X = eA_\mu$, so

$$A_\mu \xrightarrow{U(1)} A_\mu + \frac{1}{e}\partial_\mu\Lambda(x)$$

~ A is a field $\Rightarrow$ Can identify to a spin 1 boson ~

- it's tempting to
  - add the propagation of this FREE FIELD
  - add the mass of the field

$\Rightarrow$ We still have invariance of $\mathcal{L}$

$$\mathcal{L} = \underbrace{i\bar{\Psi}(x)\gamma_\mu(\partial^\mu - ieA^\mu)\Psi(x)}_{\text{Fermionic Kinetic Energy}} - \underbrace{m\bar{\Psi}(x)\Psi(x)}_{\text{Fermion Mass}} - \underbrace{\frac{1}{4}F_{\mu\nu}F^{\mu\nu}}_{\text{EM interact TERM}} + \underbrace{\frac{1}{2}m_A^2 A_\mu A^\mu}_{\text{Kinetic Energy of } A}$$

we will use $\underline{m_A = 0}$

Postulating the invariance of the lagrangian density of a FREE FERMION under the local U(1) transformation Field, we have the outcome that the FERMIONS CANNOT EXIST BY ITSELF.

There shall be a particle described by the Field A of spin 1 and ZERO MASS interacting with the Fermion.

Historically it went the other way around

$\Rightarrow$ It was found that U(1) local complies with the OBSERVATIONS.

Electro Weak Unification

The Facts

1. We have 3 families of fermions

$$\begin{pmatrix}\nu_e\\ e\end{pmatrix}\ \begin{pmatrix}\nu_\mu\\ \mu\end{pmatrix}\ \begin{pmatrix}\nu_\tau\\ \tau\end{pmatrix} \quad \text{and} \quad \begin{pmatrix}u\\ d\end{pmatrix}\ \begin{pmatrix}c\\ s\end{pmatrix}\ \begin{pmatrix}t\\ b\end{pmatrix}$$

With a well defined chirality structure $\begin{pmatrix}\nu_e\\ \ell\end{pmatrix} = \left\{ \begin{pmatrix}\nu_e\\ \ell\end{pmatrix}_L , (\nu_e)_R , \ell_R \right\}$

WHY ? Not putted into a weak isospin doublet because do NOT COUPLE with bosons of the SU(2) symmetry

2. The charged currents are mediated by charged W, dealing ONLY WITH LEFT-HANDED fermions (R-H antifermions)

3. The neutral current are mediated by Z bosons. They preserve the flavor. Coupling with BOTH L-H and R-H particles a priori

4. The couplings are universal $\Rightarrow$ same for all the quarks or lepton families.

$\rightsquigarrow$ Electro Magnetism is implicitly there. The W are charged particles

WE ARE AIMING A THEORY WHICH INCLUDES QED

What could be a good symmetry?

The CHIRALITY STRUCTURE of the fermion families is GUIDING US.
We want first a doublet for charged current interactions. We experienced several time doubled structures and

$SU(2)$ is a natural candidate

We hence introduce a new quantum number : THE WEAK ISOSPIN

$\rightarrow$ Pauli Matrices are the generators of $SU(2)$

$\rightarrow$ $SU(2)$ unlike $U(1)$ is non-abelian

- We want that neutral EM currents are unchanged under $SU(2)$ transformation.

$\Rightarrow$ WE FORM A NEW GROUP $U(1)$ and define the WEAK HYPERCHARGE of quantum number $Y = 2(Q - I_3)$ (as we did for the strong isospin)

Hypercharge:

			I	I_3	Q	Y
leptons	doublet L	ν_e	1/2	+1/2	0	-1
		e_L^-	1/2	-1/2	-1	-1
	singlet R	e_R^-	0	0	-1	-2
quarks	doublet L	u_L	1/2	+1/2	+2/3	+1/3
		d_L	1/2	-1/2	-1/3	+1/3
	singlet R	u_R	0	0	+2/3	+4/3
	singlet R	d_R	0	0	-1/3	-2/3

Generating interactions?

$$SU(2)_L \longrightarrow U_L = e^{i\frac{\vec{\alpha}\cdot\vec{\tau}}{2}}$$

$$U(1)_Y \longrightarrow U_Y = e^{iY\beta}$$

So

$$\psi_L \longrightarrow e^{iY_L\beta}\, U_L\, \psi_L$$

$$\psi_R \longrightarrow e^{iY_R\beta}\, \psi_R$$

Where τ and Y are the generators of $SU(2)$ and $U(1)$ groups, respectively.

Now let's make the LOCAL GAUGE INVARIANCE

- $\beta \to \beta(x)$: this gives rise to ONE boson B (as A in QED)
- $\alpha \to \alpha(x)$: this gives rise to THREE bosons W (W^1, W^2, W^3)

So now we have

$$\partial^\mu \psi_L \longrightarrow [\partial^\mu + ig_L B^\mu + i\frac{g_W}{2}(\vec{\tau}\cdot\vec{W})^\mu]\,\psi_L$$

$$\partial^\mu \psi_R \longrightarrow [\partial^\mu + ig_R B^\mu]\,\psi_R \qquad (*)$$

Remembering that

$$\bar{\psi}_L \psi = (\nu_L \;\; e_L)\begin{pmatrix}\nu_m \\ e_L\end{pmatrix}$$

We want to write now the lagrangian density (D^μ as the covariant derivative ENSURING local gauge invariance)

Now taking the Dirac Lagrangian

$\rightarrow \mathcal{L}_0 = i\bar{\psi}_R \gamma^\mu \partial_\mu \psi_R + i\bar{\psi}_L \gamma^\mu \partial_\mu \psi_L$ Splitted in ψ_L and ψ_R since (*)

Now let's change the derivative

$$\partial_\mu \rightarrow D_\mu = \partial_\mu + ig_L B_\mu + \frac{ig_w}{2}(\vec{\sigma}\cdot\vec{W})_\mu \quad \text{FOR} \quad LH$$

$$\partial_\mu \rightarrow D_\mu = (\partial_\mu - ig_R B_\mu) \text{ For } \psi_R \quad \text{FOR} \quad RH$$

By using this derivatives D_μ we are sure that the NEW LAGRANGIAN will looks like the INVARIANT ONE

Let's focus

$$\vec{\sigma}\cdot\vec{W}_\mu = \sigma_1 W_\mu^1 + \sigma_2 W_\mu^2 + \sigma_3 W_\mu^3$$

$$= \begin{pmatrix} 0 & W_\mu^1 \\ W_\mu^1 & 0 \end{pmatrix} + \begin{pmatrix} 0 & -iW_\mu^2 \\ iW_\mu^2 & 0 \end{pmatrix} + \begin{pmatrix} W_\mu^3 & 0 \\ 0 & -W_\mu^3 \end{pmatrix}$$

$$= \begin{pmatrix} W_\mu^3 & W_\mu^1 - iW_\mu^2 \\ W_\mu^1 + iW_\mu^2 & -W_\mu^3 \end{pmatrix}$$

CHARGED CURRENTS: NON DIAGONAL PART acting ONLY on LH states

$$W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \pm iW_\mu^2)$$

$$j_\mu^1 = g_w \bar{\psi}_L \gamma_\mu \frac{\sigma_1}{2} \psi_L \quad \text{and} \quad j_\mu^2 = g_w \bar{\psi}_L \gamma_\mu \frac{\sigma_2}{2} \psi_L$$

let's define

$$j_\mu^\pm = \frac{1}{\sqrt{2}}(j_\mu^1 \pm i\, j_\mu^2) = \frac{g_w}{2\sqrt{2}} \bar{\psi}_L \gamma_\mu (\sigma_1 \pm i\sigma_2) \psi_L$$

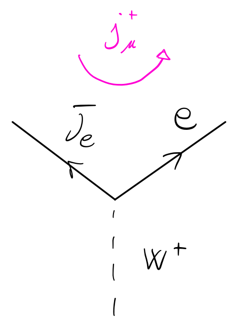
$$= \frac{g_w}{\sqrt{2}} \bar{\psi}_L \gamma_\mu \sigma^\pm \psi_L$$

with $\quad \sigma^{\pm} = \frac{1}{2}(\sigma^1 \pm i\sigma^2) \qquad$ (Example $\sigma^{+} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$

Application

$$\Psi_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}; \qquad j^{+}_{\mu} = \frac{g_w}{\sqrt{2}} (\bar{\nu}_L \ \ \bar{e}_L)\, \gamma_\mu \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$$

$$= \frac{g_w}{\sqrt{2}} \bar{\nu}_L \gamma_\mu e_L$$



but

$$\begin{cases} e_L = P_L e = \frac{1}{2}(1-\gamma^5)\, e \\ \nu_L = P_L \nu \\ \bar{\nu}_L = (\overline{P_L \nu}) = (P_L \nu)^{\dagger} \gamma^0 = \nu^{\dagger} P_L^{\dagger} \gamma^0 = \nu^{\dagger} \frac{1}{2}(1-\gamma^5)\gamma^0 = \nu^{\dagger}\gamma^0 \frac{1}{2}(1+\gamma^5) \end{cases}$$

Finally

$$\boxed{j^{+}_{\mu} = \frac{g_w}{\sqrt{2}} \bar{\nu} P_R \gamma_\mu P_L e = \frac{g_w}{\sqrt{2}} \bar{\nu} \gamma_\mu P_L^2 e = \frac{g_w}{2\sqrt{2}} \bar{\nu} \gamma_\mu (1-\gamma^5) e}$$

So genericelly

$$\mathcal{L}_{cc} = i \bar{\Psi}_L \gamma_\mu \left[i \frac{g_w}{2} \begin{pmatrix} 0 & W^1 - iW^2 \\ W^1 + iW^2 & 0 \end{pmatrix} \right] \Psi_L$$

with Ladder Operators

$$\mathcal{L}_{cc} = \frac{g_w}{\sqrt{2}} \bar{\Psi}_L \gamma_\mu \left[W_+^{\mu} \sigma_+ + W_-^{\mu} \sigma_- \right] \Psi_L$$

NEUTRAL CURRENTS

$$\mathcal{L}_{NC} = i\,\bar{\Psi}_L \gamma^\mu \left(i\frac{g'}{2} Y_L B_\mu + i g_W \frac{\sigma_3}{2} W^3_\mu \right) + i\,\bar{\Psi}_R \gamma^\mu \left(i \frac{g'}{2} Y_R B_\mu \right) \Psi_R$$

Like we did before we can identify the boson in this Lagrangian NC.

We can identify the photon as B_μ and the Z boson as W^3_μ.

$\Rightarrow$ If we do that we say Z boson only couple with LH particles

! In chapter I there is no suggestion in that! WE DO NOT WANT THAT.

We should find a way to avoid that.

Logic: B_μ and W^3_μ are NEUTRAL GAUGE BOSONS. We love Q.M.

If we have analogous neutral boson, i can MIX them in order to have phisical states.

$$\boxed{\begin{pmatrix} W^3_\mu \\ B_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix}}$$

Application

As previously let's evaluate $\mathcal{L}_{NC}$ with $\Psi_L \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$ and $\Psi_R = e_R$

$$\begin{aligned}\mathcal{L}_{NC} = &-\frac{1}{2}\bar{\nu}_L \gamma_\mu \left[(g_W C_\theta - g_L S_\theta) Z^\mu + (g_L C_\theta + g_W S_\theta) A^\mu \right] \nu_L \\ &-\frac{1}{2}\bar{e}_L \gamma_\mu \left[(-g_W C_\theta - g_L S_\theta) Z^\mu + (g_L C_\theta - g_W S_\theta) A^\mu \right] e_L \\ &-\frac{1}{2}\bar{e}_R \gamma_\mu \left[-g_R S_\theta Z^\mu + g_R C_\theta A^\mu \right] e_R\end{aligned}$$

But now we REQUIRE what we know

(1) Coupling of neutrinos with the photon is forbidden $\Rightarrow$ $g_L C_\theta + g_W S_\theta = 0$

(2) LH and RH states shall couple the same way as A field

$$\Longrightarrow \underline{g_L c_\theta - g_W s_\theta = g_R \cos\theta_W}$$

(3) The charge of the electron is e so $\underline{g' \cos_W = -e}$

Electro Week Unification

Also from the unification we will have

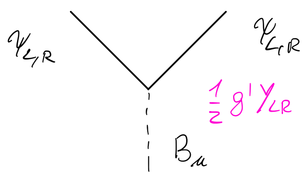
$$\alpha = \frac{e^2}{4\pi\varepsilon_0 \hbar c}$$

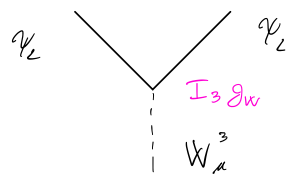
Comments:

- The couplings of electromagnetism, of weak interaction and hypercharge interaction are related
- The photon appears "naturally"
- The Z boson couplings are also defined. Let's now rewrite them by explicitly stating the weak isospin and hypercharge quantum numbers.

Z boson couplings

Let's start from W^3 and B





Couplings

$$j^B_\mu = \frac{1}{2} g' \left[\bar{\psi}_L Y_L \gamma_\mu \psi_L + \bar{\psi}_R Y_R \gamma_\mu \psi_R \right] \quad (1)$$

$$j^{W_3}_\mu = g_w \bar{\psi}_L \gamma_\mu \frac{\tau_3}{3} \psi_L \quad (2)$$

So $\quad Z_\mu = -B_\mu \sin\theta_w + W^3_\mu \cos\theta_\mu$

We know that $\begin{cases} Y_L = 2(Q - I_3) \\ Y_R = 2Q \end{cases}$ and ELECTROWEAK UNIFICATION $\begin{cases} g' = -\frac{e}{c_w} \\ g' = g_w \tan\theta_w \end{cases}$

Let's write j^Z for Z boson

$$j^Z_\mu = \frac{1}{2} g' \overset{(1)}{\left[\bar{\psi}_L 2(Q - I_3)\gamma_\mu \psi_L + \bar{\psi}_R 2Q \gamma_\mu \psi_R \right] s_w} + \overset{(2)}{\left(g_w \bar{\psi}_L I_3 \gamma_\mu \psi_L \right) c_w}$$

$$= -\frac{1}{2} g_w \frac{\cancel{s_w}}{c_w} \left[\bar{\psi}_L \left(-2 I_3 \frac{c_w^2}{\cancel{s_w}} + 2(Q - I_3)\frac{s_w^2}{\cancel{s_w}}\right) \gamma_\mu \psi_L \right] - \frac{1}{2} g_w \frac{s_w^2}{c_w} 2Q \bar{\psi}_R \gamma_\mu \psi_R$$

$$= -\frac{1}{2}\frac{g_W}{c_W}\left[2Q s_W^2 - 2I_3\right]\gamma_\mu\psi_L - \frac{1}{2}\frac{g_W}{c_W}(2Q s_W^2)\,\bar{\psi}_R\gamma_\mu\psi_R$$

$$= -\frac{1}{\cancel{2}}\frac{g_W}{c_W}\left\{\cancel{2}(Q s_W^2 - I_3)\,\bar{\psi}_L\gamma_\mu\psi_L - \cancel{2}Q s_W^2\,\bar{\psi}_R\gamma_\mu\psi_R\right]$$

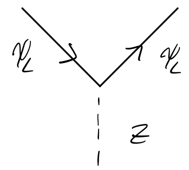
with $\begin{cases} c_L = I_3 - Q s_W^2 \\ c_R = -Q s_W^2 \end{cases}$ with $g_Z = \dfrac{g_W}{c_W}$

$$\Longrightarrow \boxed{J^Z_\mu = g_Z\left\{c_L\,\bar{\psi}_L\gamma_\mu\psi_L + c_R\,\bar{\psi}_R\gamma_\mu\psi_R\right\}}$$

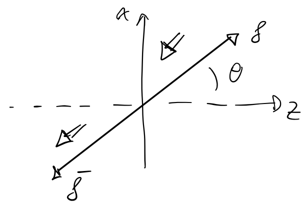
The Z width

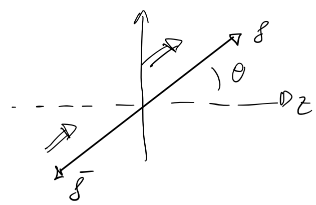
I want to understand the probability the Z decay

We have



Z couples with LH and RH states with C_L and C_R couplings





$f_L \bar{f}_L \propto g_Z C_L$ $\qquad$ $f_R \bar{f}_L \propto g_Z C_R$

Calculations

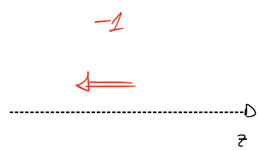
1. Basics

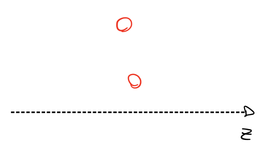
We will work in the Z rest frame

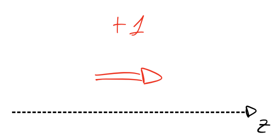
We know that Z couples to LH and RH states with C_L and C_R couplings.

2. Polarization States

$\vec{S} = \vec{1}$ and so $S_z = +1, 0, -1$







Polarization in Longitudinal

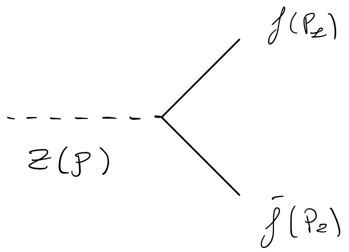
$$\varepsilon_\mu^- = \frac{1}{\sqrt{2}}(0, 1, -1, 0) \qquad \varepsilon_\mu^L = \frac{1}{m}(P_z, 0, 0, E) \qquad \varepsilon_\mu^+ = \frac{1}{\sqrt{2}}(0, 1, i, 0)$$

In the Z rest Frame $P_Z = 0$, so

$$\varepsilon^{L}_{\mu} = (0, 0, 0, 1)$$

→ Has to do with the mass.
→ Photons do not have it

3. Definitions



Spinors:

- e^- $u(p_z)$
- e^+ $u(p_z)$

SUPER WRONG, SHOULD BE v or something from QFT lectures.
Montzil said that is the same (?)

We will use

$$g_V = C_L + C_R = I_3 - 2Q S_w^2$$

$$g_A = C_L - C_R = I_3$$

$$J_\mu = \frac{g_2'}{2} \bar{\psi} \gamma_\mu (g_V - g_A \gamma^5) \psi$$

Reminder:

$$\frac{d\Gamma}{d\Omega} = \frac{p^*}{32\pi m_Z^2} |\bar{\mathcal{M}}|^2$$

$$p^* = \frac{1}{2m_Z} \sqrt{(m_Z^2 - (m_e + m_2)^2)(m_Z^2 - (m_Z - m_2)^2)}$$

4. Momenta

In the Z Reference Frame

$$P = (m_Z, 0, 0, 0)$$

$$P_Z = \left(\frac{m_Z}{2}, E\sin\theta, 0, E\cos\theta\right)$$

$$P_Z = \left(\frac{m_Z}{2}, -E\sin\theta, 0, E\cos\theta\right)$$

Computing LH interaction diagram

$$j_Z^\mu = \frac{g_Z}{2}\bar{\mu}(p_2)\gamma^\mu(g_V - g_A\gamma^5)\mu_L(p_2)$$ (Remember, it's wrong but works!)

is not maximally violated ($\neq$ neutrinos)

$$= \frac{g_Z}{2}\bar{\mu}_L(p_2)\gamma^\mu g_V\mu_L(p_2) - \frac{g_Z}{2}\bar{\mu}_L(p_2)g_A\gamma^\mu\gamma^5\mu_L(p_2)$$
$$- \frac{g_Z}{2}\bar{\mu}_L(p_2)g_A\gamma^\mu\gamma^5 P_L\mu(p_2)$$

but $\gamma^5 P_L = \gamma^5\frac{1}{2}(1-\gamma^5) = \dots = -P_L$

$$\rightarrow j_Z^{-\mu} = \frac{g_Z}{2}\bar{\mu}_L(p_2)\gamma^\mu(g_V+g_A)\mu_L(p_2)$$
$$= \frac{g_Z}{2}\bar{\mu}_L(p_2)\gamma^\mu\mu_L(p_2)(g_V+g_A)$$

QFT

$$= \frac{g_Z}{2}2E(0,-\cos\theta,-i,\sin\theta)(g_V+g_A)$$

One for each polarization

$$* \; i\mathcal{M}^- = \varepsilon_\mu^-(p)\cdot j_Z^\mu = \frac{g_Z}{2}(g_V+g_A)\frac{2\underset{=m_Z}{E}}{\sqrt{2}}(0,1,-i,0)\begin{pmatrix}0\\-\cos\theta\\-i\\\sin\theta\end{pmatrix} =$$
$$= \frac{g_Z}{2\sqrt{2}}(g_V+g_A)m_Z(-\cos\theta-1)$$

$$* \; -i\mathcal{M}^L = \varepsilon_\mu^L(p)\cdot j_Z^\mu = \frac{g_Z}{2}m_Z(g_V+g_A)(0,0,0,1)\begin{pmatrix}0\\-\cos\theta\\-i\\\sin\theta\end{pmatrix} =$$

$$* \; -i\mathcal{M}^+ = \frac{g_Z}{2\sqrt{2}}(g_V+g_A)m_Z(1-\cos\theta)$$

Squaring the amplitude to get the probability

$$|\mathcal{M}^-|^2 = \frac{g_Z}{8}(g_V+g_A)\, m_Z^2 \,(1+\cos\theta)^2$$

$$|\mathcal{M}^L|^2 = \frac{g_Z^2}{4}(g_V+g_A)\, m_Z^2 \,\sin^2\theta$$

$$|\mathcal{M}^+|^2 = \frac{g_Z}{8}(g_V+g_A)\, m_Z^2 \,(1-\cos\theta)^2$$

Hence the sum:

$$|\bar{\mathcal{M}}|^2 = \langle|\mathcal{M}|^2\rangle = \frac{1}{3}\left(|\mathcal{M}^-|^2 + |\mathcal{M}^+|^2 + |\mathcal{M}^L|^2\right)$$

AVERAGE BECAUSE WE DON'T KNOW THE INITIAL STATE

$$= |\mathcal{M}|^2 = \frac{g_Z^2}{24} m_Z^2 (g_V+g_A)^2 \left\{ \underbrace{(1+\cos\theta)^2 + 2\sin^2\theta + (1-\cos\theta)^2}_{=4} \right\}$$

$$= \frac{g_Z^2}{24} m_Z^2 (g_V+g_A)^2 \cdot 4$$

w/ $\begin{cases} g_V = C_L + C_R \\ g_A = C_L - C_R \end{cases}$ $\Rightarrow$ From this result $\boxed{|\bar{\mathcal{M}}_{LH}|^2 = \frac{2}{3} g_Z^2 m_Z^2 C_L^2}$

Calculation of RH amplitude

The only values that will change are the couplings

$$\boxed{|\mathcal{M}_{RH}|^2 = \frac{2}{3} g_Z^2 m_Z^2 C_R^2}$$

Then $$|\bar{\mathcal{M}}|^2 = |\bar{\mathcal{M}}_{LH}|^2 + |\bar{\mathcal{M}}_{RH}|^2 = \frac{2}{3} g_Z^2 m_Z^2 (c_L^2 + c_R^2)$$

and so

$$\frac{d\Gamma}{d\Omega} = \frac{m_Z}{64\pi^2 \cancel{m_Z^2}} \cdot \frac{2}{3} g_Z^2 \cancel{m_Z^2} (c_L^2 + c_R^2)$$

$$= \frac{g_Z^2}{96\pi^2} m_Z (c_L^2 + c_R^2)$$

Then integrating

$\left(\int d\Omega = 4\pi\right)$

$$\Gamma(Z \to f\bar{f}) = \frac{g_Z^2}{96\pi^2} 4\pi \; m_Z (c_L^2 + c_R^2)$$

but $\begin{cases} g_V = c_L + c_R \\ g_A = c_L - c_R \end{cases} \to c_L^2 + c_R^2 = \frac{1}{4}\left[(g_V + g_A)^2 + (g_V - g_A)^2\right]$

$$= \frac{1}{2}(g_V^2 + g_A^2)$$

$$\implies \boxed{\Gamma(Z \to f\bar{f}) = \frac{g_Z^2}{48\pi} m_Z (g_V^2 + g_A^2)}$$

$$g_Z^2 = \frac{g_W^2}{\cos^2\theta_W} \qquad G_F = \frac{\sqrt{2}}{8}\frac{g_W^2}{m_W^2} \qquad g_W^2 = \frac{8 m_W^2 G_F}{\sqrt{2}}$$

After experiments:

$g_W^2 = 0.42612$ NOT WEAK $\qquad g_Z^2 = 0.55\ell$

For neutrinos $g_V^2 = g_A^2 = \frac{1}{4}$, $g_V^2 + g_A^2 = \frac{1}{2}$

$$\Gamma(Z \to \nu\nu) = 161.5 \text{ MeV}$$
$$= 21\%$$

Outlook:

$$\mathcal{L}_{EW} = \underbrace{i\bar{f}\gamma_\mu\partial^\mu f}_{\text{Fermionic Kinematic}} + \underbrace{e\bar{f}(x)\gamma_\mu Q A^\mu f}_{\text{Gauging Theory (ELECTROMAGNETISM) at 1° order}} + \underbrace{\frac{g_W}{\sqrt{2}}[\bar{\nu}_L\gamma_\mu e_L W^\mu_- + \bar{e}_L\gamma_\mu\nu_L W^\mu_+] + g_Z[\bar{f}\gamma_\mu(g_V - g_A\gamma^5)fZ^\mu_0]}_{\text{Big succes!}}$$

Since there is a boson, we can add kinematic terms for boson propagation.

Let's first play around with kinetic term of the $\mathcal{L}$

We must start from the COVARIANT derivative

$$D_\mu = \partial_\mu + iQA_\mu$$

Let's remind the transformation of A_μ field:

$$A_\mu \to A'_\mu = A_\mu - \frac{1}{Q}\partial_\mu\Lambda(x)$$

Now let's do this calculation

$$\begin{aligned}
[D_\mu, D_\nu] &= (\partial_\mu + iQA_\mu)(\partial_\nu + iQA_\nu) - (\partial_\nu + iQA_\nu)(\partial_\mu + iQA_\mu) \\
&= \cancel{\partial_\mu\partial_\nu} + iQ\cancel{(A_\mu\partial_\nu)} + iQ\partial_\mu A_\nu - Q^2A_\mu A_\nu - \cancel{\partial_\nu\partial_\mu} - i\cancel{QA_\nu\partial_\mu} - iQ\partial_\nu A_\mu + Q^2A_\nu A_\mu \\
&= iQ(\partial_\mu A_\nu - \partial_\nu A_\mu) - Q^2(A_\mu A_\nu - A_\nu A_\mu) \\
&= iQ(\partial_\mu A_\nu - \partial_\nu A_\mu) - Q^2\underbrace{[A_\mu, A_\nu]}_{=0} \\
&= iQF_{\mu\nu}
\end{aligned}$$

U(1) Abelian Group $\quad A_\mu A_\nu = A_\nu A_\mu$

it's the strenght tensor

Now, as soon as we do the same for $W^{\mu\nu} = \partial^\mu W^\nu - \partial^\nu W^\mu - \underline{ig[W^\mu, W^\nu]}$

$\Rightarrow$ You cannot DELETE THE LAST TERM

But why? In $U(1)$ group elements commute with each other (they are complex number). Non abelian groups member are matrices for example.

So when computing the $W^{\mu\nu}$ strength tensor we will get terms like $\varepsilon^{abc} W^b_\mu W^c_\nu$ that contains the ε^{abc}, that is the same ε^{abc} that appears in the commutation relation $[T_i, T_j] = 2i\,\varepsilon_{ijk} T_k$. That's why they don't cancel out.

Easy way to answer: To compute $W^{\mu\nu}$ you need the COVARIANT DERIVATIVE:

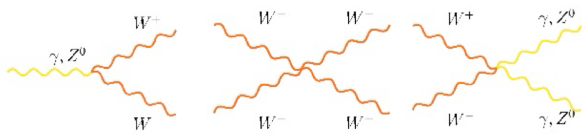
$$D_\mu = \partial_\mu - ig W^a_\mu \frac{\tau^a}{2}$$ $\rightarrow$ The τ do not commute

$\Rightarrow$ END.

And if you define $B^{\mu\nu} = \partial^\mu B^\nu - \partial^\nu B^\mu$, it's the same as A^μ we did before, NO COMMUTATOR.

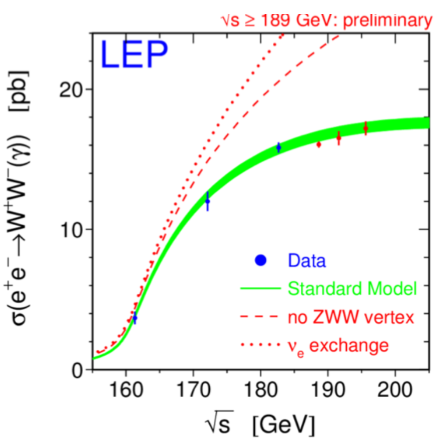
Then the Lagrangian kinetic term $$\boxed{\mathcal{L}_K = -\frac{1}{4} B^{\mu\nu} B_{\mu\nu} - \frac{1}{4} W^{\mu\nu} W_{\mu\nu}}$$

So the interaction allowed are



We checked the physics

There is WWW vertex



$\rightarrow$ There is INTERFERENCE:

$$\mathcal{M} = |\underbrace{\mathcal{M}_{ZWW} + \mathcal{M}_{NO\,ZWW}}_{\text{same ending state}}|^2$$

Spontaneus Symmetry Breaking

Motivations:

$SU(2) \otimes U(1)$ is a good theory

✓ Weak and electromagnetic gauge bosons simultaneously described.

✓ Couplings are universal

✓ Good chiral properties.

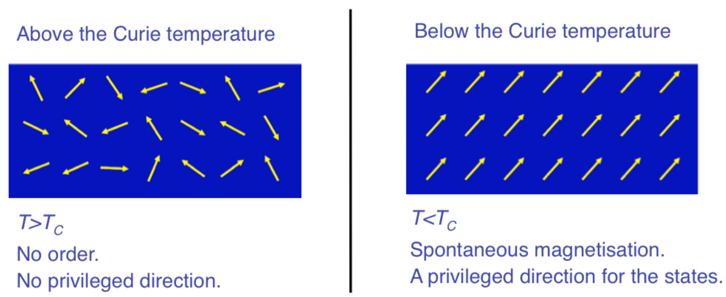
Fails indeed in:

✗ Z and W masses are forbidden by local gauge invariance

✗ Fermion Masses are forbidden for the same reason.

Example 1 of Spontaneus Symmetry Breaking.

Ferromagnet



At T_C, there is spontaneous symmetry breaking. Above the Curie temperature the symmetry is hidden. We'd like to have the same for $SU(2)_L \otimes U(1)_Y$.

Example 2: a photon in plasma

Let's consider an EM wave of energy E and pulsation ω, propagating in a plasma (pulsation ω_p and energy E_0)

$\longrightarrow$ The propagation occurs if $E > E_0$

- The refractive index would be given by the dispersion relation

$$n^2 = 1 - \frac{\omega_p^2}{\omega^2}$$

with group velocity

$$v_G \cdot v_P = c^2$$

Now $$v_G = \frac{c}{n + \omega \frac{\partial n}{\partial \omega}}$$

higher order of $v_G = \frac{c}{n}$

Let's see why $$n = \frac{(\omega^2 - \omega_p^2)^{1/2}}{\omega}$$

$$\frac{\partial n}{\partial \omega} = \frac{\frac{1}{2}(2\omega)(\omega^2 - \omega_p^2)^{-1/2} \cdot \omega - (\omega^2 - \omega_p^2)^{1/2}}{\omega^2}$$

$$= \frac{1}{n\omega} - \frac{n}{\omega} = \frac{(1 - n^2)}{n\omega^2}$$

Going back to group velocity $$v_G = \frac{c}{n + \omega \frac{(1-n^2)}{\omega^2}} = \frac{c n \omega}{\cancel{n^2\omega} + \omega - \cancel{n^2\omega}} = c n$$

$\Longrightarrow$ Must be $n < 1$ if v_G

Now $$v_G = c n = \frac{c(\omega^2 - \omega_p^2)^{1/2}}{\omega}$$

and $E = \hbar\omega$; $E_0 = \hbar \omega_p$

We have $v_G = \frac{c(E^2 - E_0^2)^{1/2}}{E}$

$$E^2 - E_0^2 = E^2 \frac{v_G^2}{c^2} \iff \boxed{E^2\left(1 - \frac{v_G^2}{c^2}\right) = E_0^2}$$

Let's note $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$

$$E = \gamma E_0$$

Let $E_0 = m_0 c^2$, $E = \gamma m^2 c^2$

So: By PROPAGATING IN THE MEDIUM IT'S LIKE γ is acquiring mass

One would like to fill the vacuum with a field interacting with the intermediate bosons and fermions.

Which kind of field?

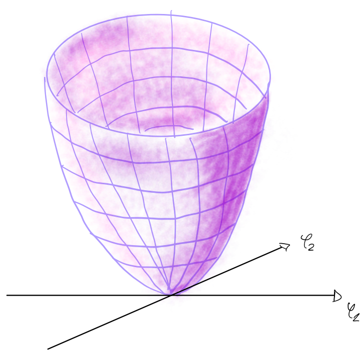
The simplest start will be SCALAR PARTICLE

$$\rightarrow \mathcal{L} = \partial^\mu \varphi^\dagger \partial_\mu \varphi - V(\varphi)$$

The POTENTIAL V contains either mass and interaction terms:

$$V(\varphi) = \mu^2 \varphi^\dagger \varphi + \lambda(\varphi^\dagger \varphi)^2$$ which are invariant under $U(1)$

We are searching for the minimum of V



- Trivial Solution $\varphi_0 = 0$
- Parameter $\lambda > 0$ and $\mu^2 > 0$
- Mass of the field $m(\varphi) = \mu^2$

Why the plot is in 2D? A SCALAR PARTICLE COULD BE REAL OR COMPLEX. If it's COMPLEX then the $\varphi = \frac{1}{\sqrt{2}}(\varphi_1 + i\varphi_2)$, with $\varphi^2 = \frac{1}{2}\varphi_1^2 + \varphi_2^2$

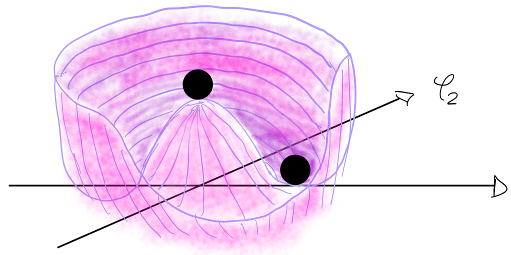
Non trivial solution $\varphi_0 \neq 0$

Parameter $\lambda > 0$ and $\mu^2 < 0$

Degenerate solutions:

$$|\varphi_0| = \sqrt{\frac{-\mu^2}{2\lambda}} = \frac{v}{\sqrt{2}} > 0$$

$$V(\varphi_0) = -\frac{1}{4}\lambda v^4$$



Remeber that in here we still have $(\partial\varphi\partial\varphi^*)$ that is not $U(1)$ invariant. We still have to impose the local symmetry.

Non trivial minimum

$\frac{\partial V}{\partial \varphi} = 0$ and we are not interested in $\begin{cases} \varphi = 0 \\ 2\mu^2\varphi + 4\lambda\varphi^3 = 0 \end{cases}$

So $\varphi_0^2 = \frac{-2\mu^2}{4\lambda}$, $\varphi_0 = \sqrt{\frac{-\mu^2}{2\lambda}} = \frac{v}{\sqrt{2}} > 0$ → MUST BE $\mu^2 < 0$ $\lambda > 0$

⇒ $\mu^2 \varphi\varphi$ it's NOT THE MASS TERM

Let's give some definition

$$\varphi_0 = \langle 0 | \varphi | 0 \rangle = \frac{v}{\sqrt{2}}$$

v.e.v it's the vacuum expectation number
IT'S THE VALUE OF THE FIELD AT MINIMUM POTENTIAL

SYMMETRY BREAKING!

EVEN IF WE STARTED WITH A SYMMETRICAL LAGRANGIAN we ended up with a SOLUTION TO THE EQUATION OF MOTION THAT DOES NOT CONTAIN THE INFORMATION OF THE SYMMETRY ⇒ Does not RESPECT THE SYMMETRY

Why I'm saying "solution to the equation of motion"?

Minimizing the potential actually means solving the E.O.M:

$$\frac{\partial \mathcal{L}}{\partial \varphi} - \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi)} \right) = -\frac{\partial V}{\partial \varphi} - \frac{1}{2}\partial_\mu \partial^\mu \varphi = 0$$

solution for $\partial_\mu \varphi = 0$:

$$\frac{\partial V}{\partial \varphi} = -\mu^2\varphi + \lambda\varphi^3 = 0 \qquad \varphi = \begin{cases} 0 \\ +v \\ -v \end{cases}$$

What's the value of the potential at the minimum?

$$V(\varphi) = \mu^2 \varphi_0^2 + \lambda \varphi_0^4$$
$$= \mu^2 \frac{-\mu^2}{2\lambda} + \lambda \frac{v^4}{4}$$
$$= -\lambda v^2 \cdot \frac{\lambda v^2}{2\lambda} + \frac{\lambda v^4}{4}$$
$$= -\frac{1}{2}\lambda v^4 + \lambda\frac{v^4}{4} = -\frac{1}{4}\lambda v^4$$

What's the particle content of this field?

Why particle content? The EXITATIONS of field are describing the particle states. They can be obtained by considering a perturbation of the field φ around THE VACUUM STATE.

Let's fluctuate locally the field φ to let it be REAL FIELD.

$$\varphi(x) = \underbrace{\frac{1}{\sqrt{2}}}_{\varphi_0}(v+\eta(x))$$

where $\eta(x)$ it's a scalar field particle real for the sake of semplification.

$$\longrightarrow \quad \mathcal{L} = \partial^\mu \varphi^\dagger \partial_\mu \varphi - V(\varphi) \quad \longleftarrow$$

Let's start with the potential term of the lagrangian (omitting the minus sign)

$$\mathcal{L}_V = \mu^2\varphi^2 + \lambda\varphi^4 = \frac{\mu^2}{2}(v+\eta)^\dagger(v+\eta) + \frac{\lambda}{4}[(v+\eta)^\dagger(v+\eta)]^2$$

$$= \frac{\mu^2}{2}(v+\eta)^2 + \frac{\lambda}{4}(v+\eta)^4$$

Let's evaluate at minimum $\mu^2 = -\lambda v^2$

because $\varphi_0 = \sqrt{\frac{-\mu^2}{\lambda}} = \pm v$

$$= -\frac{\lambda v^2}{2}(v^2 + \cancel{2\eta v} + \eta^2) + \frac{\lambda}{4}(v^4 + \cancel{4v^3\eta} + 6v^2\eta^2 + 4v\eta^3 + \eta^4)$$

$$= \underbrace{-\frac{1}{4}\lambda v^4}_{\text{CONSTANT}} + \underbrace{\lambda v^2\eta^2}_{\text{MASS TERM}} + \lambda v\eta^3 + \frac{1}{4}\lambda\eta^4$$

Now we will impose the local gauge invariance

$\rightarrow$ We introduce the SU(2) doublet $\varphi = \begin{pmatrix}\varphi^+ \\ \varphi^0\end{pmatrix}$ change of charge $+1$

Quantum numbers:

$$I_3(\varphi^+) = \frac{1}{2} \qquad I_3(\varphi^0) = -\frac{1}{2}$$

$$Y(\varphi^+) = 2(Q - I_3) = +1 \qquad Y(\varphi_0) = +1$$

To comply with $SU(2)_L \times U(1)_Y$, let's introduce the doublet $\varphi = \begin{pmatrix} \varphi^{(+)} \\ \varphi^{(0)} \end{pmatrix}$

The Lagrangian for these scalar fields

$$\mathcal{L}_H = (D_\mu \varphi)^\dagger (D_\mu \varphi) - \mu^2 \varphi^\dagger \varphi - \lambda (\varphi^\dagger \varphi)^2$$

$$\text{w/} \quad D_\mu \varphi = \partial_\mu \varphi - i \frac{g_W}{2} W_\mu^i \tau_i - i \frac{g'}{2} \underbrace{Y_\varphi}_{=1} B_\mu \varphi$$

From last Lecture

Let's evaluate $(D_\mu \varphi)^\dagger (D_\mu \varphi)$

Considering only the gauge field, we must force the charged field to be 0 to preserve $U(1)_{EM}$ invar.

- $$D_\mu \varphi = -i \left[\frac{g_W}{2} \begin{pmatrix} W_{3\mu} & W_{1\mu} - i W_{2\mu} \\ W_{1\mu} + i W_{2\mu} & -W_{3\mu} \end{pmatrix} + \frac{g_W}{2} \frac{S_W}{C_W} B_\mu \right] \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v+H \end{pmatrix}$$

$$\text{w/} \quad g' = g_W \tan\theta_W$$

$$= -i \frac{g_W}{2\sqrt{2}} \frac{(v+H)}{C_W} \begin{pmatrix} W_{3\mu} C_W + S_W B_\mu & C_\mu (W_{1\mu} - i W_{2\mu}) \\ -C_W (W_{1\mu} + i W_{2\mu}) & -C_W W_{3\mu} + S_W B_\mu \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$= \frac{-i g_W}{2\sqrt{2}} \frac{(v+H)}{C_W} \begin{pmatrix} C_W (W_{1\mu} - i W_{2\mu}) \\ -C_W W_{3\mu} + S_W B_\mu \end{pmatrix}$$

- $$(D_\mu \varphi)^\dagger = \frac{i g_W}{2\sqrt{2}} \frac{(v+H)^\dagger}{C_W} \left(C_W (W_1^{*\mu} + i W_2^{*\mu}) \,,\, -C_W W_3^{*\mu} + S_W B^{*\mu} \right)$$

So

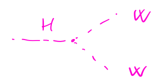
$$(D_\mu \varphi)^\dagger (D_\mu \varphi) = \frac{g_w^2}{8 c_w} (v+H)^2 \Big\{ c_w^2 (W_1^{*\mu} + i W_2^{*\mu})(W_{1\mu} - i W_{2\mu}) \quad + $$
$$(-c_w W_3^{*\mu} + s_w B^\mu)(-c_w W_{3\mu} + s_w B_\mu) \Big\}$$

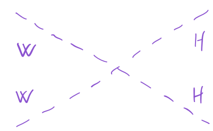
Notation:

$$W_\mu^+ = \frac{1}{\sqrt{2}} (W_{1\mu} + i W_{2\mu}) \quad \text{and} \quad (W^+)^\dagger = W^-$$

Let's consider the First term of $(D_\mu \varphi)^\dagger (D_\mu \varphi)$ (Here $W^\mu = W^{*\mu}$)

$$\frac{g_w^2}{4 c_w^2} (v+H)^2 \left\{ c_w^2 W^\mu W_\mu \right\} = \underbrace{\frac{g_w^2}{4 c_w}}_{m_W^2} v^2 W^\mu W_\mu + \underbrace{\frac{1}{2} g_w^2 v H W^\mu W_\mu}_{} + \underbrace{\frac{1}{4} g_w^2 H^2 W^\mu W_\mu}_{}$$





$$\downarrow$$

$$m_W = \frac{1}{2} g_w v = M_Z \cos\theta_W$$

Second Term

$$\frac{g_w^2}{8 c_w^2} (v+H)^2 \left\{ + c_w^2 W_3^\mu W_{3\mu} - s_w c_w W_3^\mu B_\mu - s_w c_w B^\mu W_{3\mu} + s_w^2 B^\mu B_\mu \right\}$$

Remember that

$$\begin{pmatrix} W_3^\mu \\ B_\mu \end{pmatrix} = \begin{pmatrix} c_w & s_w \\ -s_w & c_w \end{pmatrix} \begin{pmatrix} Z^\mu \\ A^\mu \end{pmatrix}$$

So

$$\frac{g_w^2}{8 c_w^2} (v+H)^2 \Big\{ c_w^2 (c_w Z^\mu + s_w A^\mu)(c_w Z_\mu + s_w A_\mu) - s_w c_w (c_w Z^\mu + s_w A^\mu)(-s_w Z_\mu + c_w A_\mu)$$
$$- s_w c_w (-s_w Z^\mu + c_w A^\mu)(c_w Z_\mu + s_w A_\mu) + s_w^2 (-s_w Z^\mu + c_w A^\mu)(-s_w Z_\mu + c_w A_\mu) \Big\}$$

$\Rightarrow$ All terms $Z^\mu A_\mu$ and $A^\mu Z_\mu$ are canceling out

$\Longrightarrow$ Let's check the $A^\mu A_\mu$ term

$$A^\mu A_\mu \left(c_W^2 s_W^2 - c_W^2 s_W^2 - s_W^2 c_W^2 + s_W^2 c_W^2\right) \Rightarrow$$ The photon remain massless

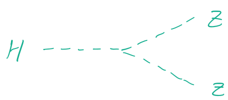
$\Longrightarrow$ Now the $Z^\mu Z_\mu$ term

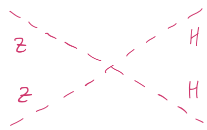
$$Z^\mu Z_\mu \left(c_W^4 + s_W^4 + 2 s_W^2 c_W^2\right) \frac{g_W^2}{8c_W^2} (v+H)^2 = \frac{g_W^2}{8c_W^2} (v+H)^2 Z^\mu Z_\mu$$

So

$$\frac{g_W}{8c_W^2} v^2 Z^\mu Z_\mu + \frac{1}{4}\frac{g_W}{c_W^2} v H Z^\mu Z_\mu + \frac{1}{8}\frac{g_W^2}{c_W^2} H^2 Z^\mu Z_\mu$$

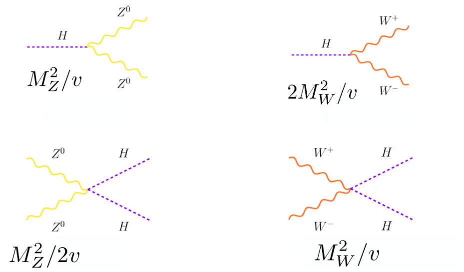
$$m_Z = \frac{1}{2}\frac{g_W v}{c_W} = \frac{m_W}{c_W}$$





Super hight energy required

Let's do a recap of interactions between H, Z, W

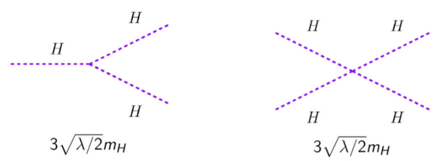


These couplings are pretty well predicted. The v value can be extracted from low energy measurements (muon lifetime for instance).

$$\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{8M_W^2} = \frac{1}{2v^2}$$
$$v = (G_F\sqrt{2})^{-\frac{1}{2}} = 246 \text{ GeV}.$$

Also the Lagrangian exhibits terms uniquely involving the Higgs boson.

$$\mathcal{L}_H = \frac{1}{2}\partial_\mu H \partial^\mu H - \frac{1}{2}m_H^2 H^2 - \frac{m_H^2}{2v}H^3 - \frac{m_H^2}{8v^2}H^4$$



Probing these couplings is a kind of graal. They are defining the shape of the potential. No chance at LHC.

with a Higgs of mass $m_H = \sqrt{2}\mu^2 = \sqrt{\lambda/2}v$

Electroweak symmetry breaking: FERMIONS?

The gauge invariance allows Yukawa couplings between fermions and scalar:

$$\mathcal{L}_Y = (\bar{q}_u \; \bar{q}_d)_L \left[c^{(d)} \begin{pmatrix} \varphi^{(+)} \\ \varphi^{(0)} \end{pmatrix} (q_d)_R + c^{(u)} \begin{pmatrix} \varphi^{(0)\dagger} \\ -\varphi^{(+)\dagger} \end{pmatrix} (q_u)_R \right] + (\bar{\nu}_e \; \bar{\ell})_L \, c^{(\ell)} \begin{pmatrix} \varphi^{(+)} \\ \varphi^{(0)} \end{pmatrix} \ell_R + h.c.$$

Calculation illustration

Before the EWSyBr:

$$\mathcal{L}_Y = G_Y^e \left[(\bar{\nu}_e, \bar{e})_L \begin{pmatrix} \varphi^+ \\ \varphi^{0\dagger} \end{pmatrix} e_R + \bar{e}_R \, (\varphi^-, \varphi^0) \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \right]$$

$\ell_R + \varphi_0 \longrightarrow \ell_R$ $\qquad$ φ couples RH and LH particles

	ℓ_R	φ_0	ℓ_R
Y	-2	1	-1
I_3	0	$-\frac{1}{2}$	$-\frac{1}{2}$

Let's break the EWSy: $\quad \varphi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H \end{pmatrix}$

$$\mathcal{L}_Y = G_Y^e \, (\bar{\nu}_e, \bar{e})_L \, \frac{(v+H)}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix} e_R$$

$$= \frac{G_Y^e}{\sqrt{2}} (v + H) \, \bar{e}_L e_R$$

This is mass term is $\frac{G_Y^e}{\sqrt{2}} v = m_e$

The mass matrices

Now that the mass is defined, one would like to look at the fermion mass matrices.

The 3 generations are identical in the theory except for the masses:

$$\mathcal{L}_Y = -\left(1+\frac{H}{v}\right)\sum_{i,j}\left[(\bar{q}'_d)^i_L (M_d)_{ij} (q'_d)^j_R + (\bar{q}'_u)^i_L (M_u)_{ij} (q'_u)^j_R + (\bar{\ell}')^i_L (M_e)_{ij} (\ell')^j_R + h.c.\right]$$

The mass states are obtained by diagonalizing the mass matrix:

$$M_f = H_f U_f = V_f^\dagger m_f V_f U_f$$

Here m is diagonal, U and V are unitary.

The fermion states are redefined according to:

$$(q_d)_L = V_d (q'_d)_L \qquad (q_u)_L = V_u (q'_u)_L \qquad \ell_L = V_\ell \ell'_L$$

$$(q_d)_R = V_d U_d (q'_d)_R \qquad (q_u)_R = V_u U_u (q'_u)_R \qquad \ell_R = V_L U_L \ell'_R$$

We recovered a mass matrix that is NOT diagonal

Matrix V

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

- $VV^\dagger = \mathbb{1}$ $\qquad V_{ij} \in \mathbb{C}$

$\rightarrow |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$

$\rightarrow V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0$

$\vdots$

3 constrain on real parameter

3 phase constrain

A priori 18 free parameter : 9 real, 9 phases

$\#$ real parameters: $9 - 6 = 3$

$\#$ phase parameters: $9 - 3 - 5 = 1$

5 since we can redefine each quark field phase (6) up to a single phase (1), $5 = 6 - 1$.

→ There are then couplings of the H boson w/ Fermions, proportionally to the square of the mass of the Fermions.

Top quark leads the game but can't be produced on-shell.