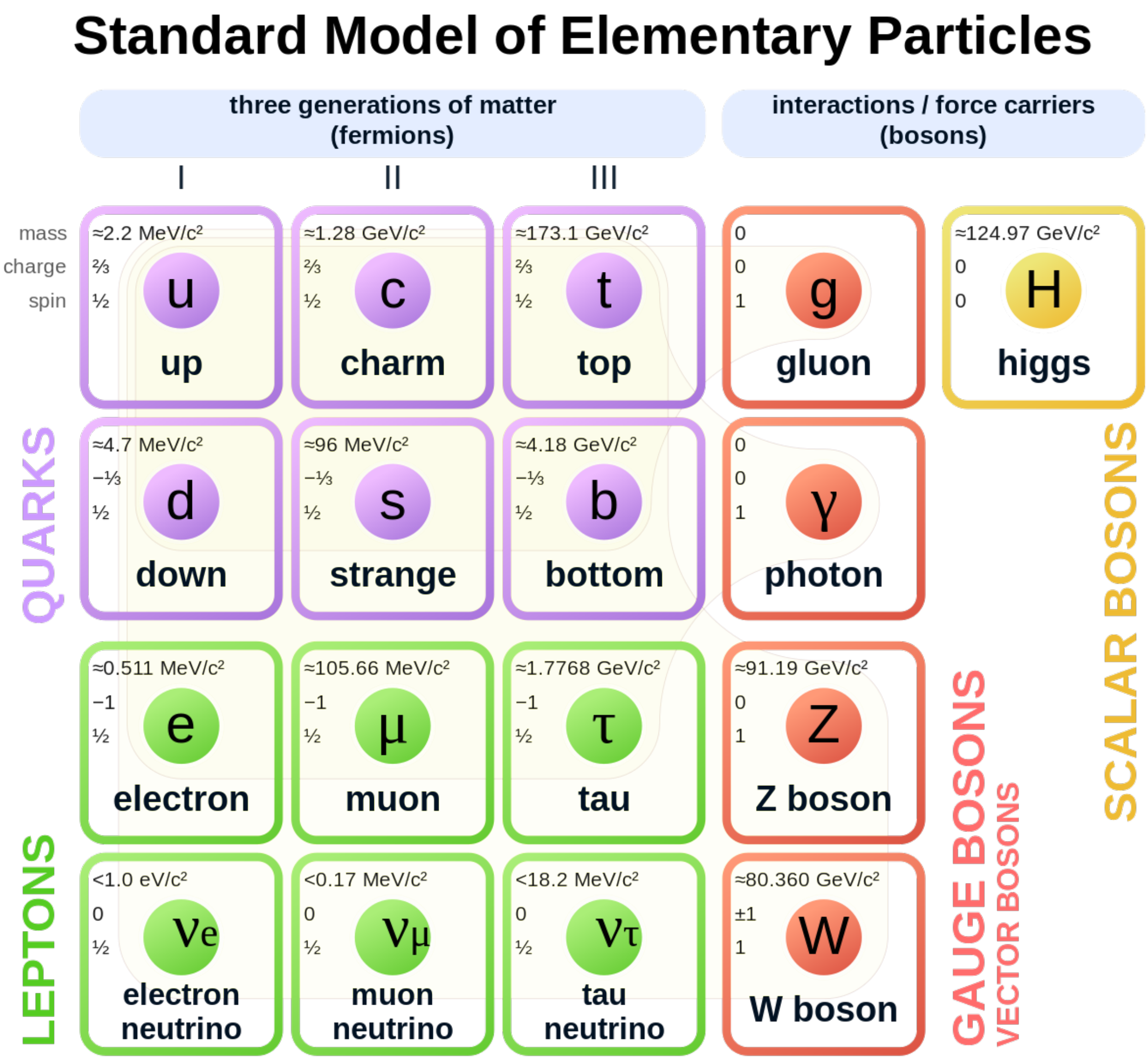
Introduction to Particle Physics



Chapter 1 - QED, Dirac, ...

I - Introduction

Theory Landscape

SR, QM $\xrightarrow{+}$ RQM $\longrightarrow$ QFT

based on Dirac Equation — Standard Model

Is missing GR

Natural Unit

$$\hbar = 1,055 \times 10^{-34}\ Js$$

$$c = 2,9979 \times 10^{8}\ m/s$$

N. U.

$$\hbar = c = 1$$

So it means

$$[\hbar] = E \times T^{-1} \equiv 1 \rightarrow E = T^{-1} \longrightarrow \text{Fourier Transformation}$$

$$[c] = L \times T^{-1} = 1 \rightarrow L = T \longrightarrow \text{Space and Time are correlated}$$

So

$$1\ m = 5,07\ 10^{6}\ eV^{-1}$$

$$1\ s = 1,52\ 10^{15}\ eV^{-1}$$

$$\hbar c = 197\ MeV \cdot Fm$$

II - Particle Decay and Cross Section

1. Decay Equation

$$N(t) = N(t=0)\, e^{-\lambda t}$$

λ : decay constant

$\lambda = \frac{1}{\tau}$ $\quad$ τ : lifetime

The lifetime is Δt, From Heisenberg Principle $\Delta t \Delta E \simeq \hbar$, so i can associate an Energy

$$\Delta E \simeq \frac{\hbar}{\Delta t}$$

Introduce the Transition Rate Γ, so $\boxed{\Gamma = \frac{1}{\tau}}$

2. Description of decay states in QM

Consider a q.m. state produced at $t=0$ with energy E and lifetime τ

$$\psi(t) = \psi(t=0)\, e^{-iEt}$$

can E be a real quantity? **NO** otherwise we would have the amplitude that remain unchanged!

$$|\psi(t)|^2 = |\psi(0)|^2 \quad \Rightarrow \text{It's not a decay particle,}$$

Insted we write $E = E_0 - \dfrac{i}{2\tau} = E_0 - i\dfrac{\Gamma}{2}$ with

E_0: energy of the particle at the resonance

Γ: transition rate OR Particle Width

So $\boxed{\psi(t) = \psi(0)\, e^{-iE_0 t}\, e^{-\frac{\Gamma t}{2}}}$ and $|\psi(t)|^2 = |\psi(0)|^2 e^{-\Gamma t} \neq |\psi(0)|^2$

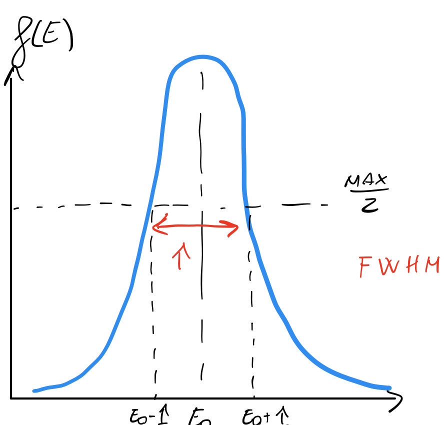
$\Rightarrow$ The probability density is decreasing exponentially (as required)

Sometimes we are interested in the Energy $\Rightarrow$ $\psi(t) \xrightarrow{\text{Fourier}} \tilde{\psi}(E)$

We will have $$\tilde{\psi}(E) = \frac{i\psi(0)}{\sqrt{2\pi}} \frac{1}{(E-E_0) + i\frac{\Gamma}{2}}$$

We have for Fourier Proprierties $|\psi(t)|^2 = |\tilde{\psi}(E)|^2 \sim$ amplitude $|A|^2$

$$|\psi(E)|^2 = |\psi(E)|^2_{MAX} \times \boxed{\frac{\Gamma^2/4}{(E-E_0)^2 + \frac{\Gamma^2}{4}}}$$ Breit-Wigner Function



$$f(E) = \frac{\Gamma^2/4}{(E-E_0)^2 + \Gamma^2/4} = \left|\frac{\Gamma/2}{(E-E_0) + i\frac{\Gamma}{2}}\right|^2$$

- Definition: OFF-Shell $\rightarrow$ whithin the Γ mass
  ON-shell $\rightarrow$ outside the Γ mass

Example: Process $A + B \rightarrow$ Resonance $\rightarrow A + B$

$$\pi + p \rightarrow \Delta(1232\ \text{MeV}) \rightarrow \pi + p$$

The cross section of this process is proportional to $|\tilde{\psi}(E)|^2$

$$\sigma(E) = \sigma_{MAX} \frac{\Gamma_{AB}^2/4}{(E-E_0)^2 + \frac{\Gamma^2}{4}}$$

with Γ_{AB} : partial with Resonance $\rightarrow A+B$
Γ : Full with

3. Fermi – Golden Rule

In non-relativistic quantum mechanics, transition state from an initial state $|i\rangle$ to a final state $|f\rangle$ are expressed by

$$\boxed{\Gamma_{fi} = 2\pi \, |T_{fi}|^2 \, \rho(E_i)} \qquad \text{Golden Rule}$$

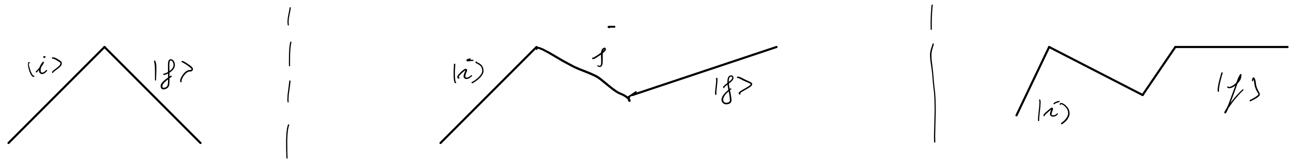
- T_{fi} : Transiction Matrix
- ρ : Energy Density of states

Hamiltonian $\hat{H} = \hat{H}_{free} + \hat{H}_{interaction}$ so T_{fi} is determined by $\hat{H}_{int}$:

$$T_{fi} = \langle f | \hat{H}_{int} | i \rangle + \sum_{j \neq i} \frac{\langle f | \hat{H}_{int} | j \rangle \langle j | \hat{H}_{int} | i \rangle}{E_i - E_j} + \dots$$

In the limit where perturbation is weak

A picture



All this is computable because we consider perturbative interactions.
But it's not Relativistic-Invariant.

4. Relativistic Particle Decay

For relativistic particle the transition matrix T_{fi} is replaced by the matrix element $\mathcal{M}_{fi}$, that is Lorentz invariant

For a general process $a + b + \dots \rightarrow 1 + 2 + \dots$

$$\mathcal{M}_{fi} = \langle \psi_1 \psi_2 \psi_3 \dots | \hat{H}_{int} | \psi_a \psi_b \psi_c \dots \rangle = T_{fi} \left(2E_1 \cdot 2E_2 \cdots \times 2E_a \cdot 2E_b \right)^{1/2}$$

In general particle decay in more than 1 mode

$$\tau^- \to e^- \bar{\nu}_e \, \nu_\tau \qquad BR = 0.17$$
$$\tau^- \to \mu^- \bar{\nu}_\mu \, \nu_\tau$$
$$\tau \to \nu_\tau + \text{hadrons}$$

The total transition rate is Γ (total width)

$$\Gamma = \sum_{i=1}^{N} \Gamma_j \qquad N: \text{ number of possible decay mode}$$

<u>Branching Ratio</u>

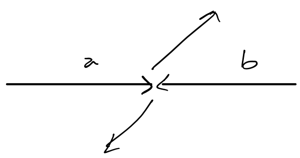
$$BR_j = \frac{\Gamma_j}{\Gamma}$$

5) Interaction cross-section

Interaction Rate = flux of incident particle × number of target particle × Cross Section

[Cross Section] the dimension is an area, expressed in barn $\quad 1\,\text{barn} = 100\ \text{fm}^2 = 100 \times (10^{-15})^2 \doteq 10^{-28}\ \text{m}^2$

Example of calculation:



$$\sigma = \frac{(2\pi)^{-2}}{F} \int |M_{fi}|^2 \, \delta(E_a + E_b - E_1 - E_2) \, \delta^3(\vec{p}_a + \vec{p}_b - \vec{p}_1 - \vec{p}_2) \, \frac{d^2 \vec{p}_1}{2E_1} \times \frac{d^3 \vec{p}_2}{2E_2}$$

III. Dirac Equation

1. Reminders

Schrödinger is QM non-relativistic equation

$$E = \frac{\vec{p}^2}{2m} \qquad \text{Operators} \qquad \hat{\vec{p}} = \frac{\hbar}{i}\vec{\nabla}$$

$$\hat{E} = i\hbar \frac{\partial}{\partial t}$$

$$\Rightarrow \quad i\hbar \frac{\partial}{\partial t}\psi(\vec{R},t) = -\frac{\hbar^2}{2m}\vec{\nabla}^2\psi + V\psi$$

Relativistic equation

$$E^2 = p^2c^2 + m^2c^4$$

$$\boxed{\left(\vec{\nabla} - \frac{1}{c^2}\frac{\partial}{\partial t^2}\right)\psi = \frac{m^2c^2}{\hbar}\psi} \qquad \text{Klein Gordon Equation}$$

Solutions:
Klein-Gordon equation has plane wave solutions

$$\psi(\vec{x},t) = N e^{i(\vec{p}\cdot\vec{x} - Et)}$$

Problems:

1. You can have negative energy solution
2. Negative probability density $\rho = \psi^*\psi$
3. Does not describe kinematics of electron

It's used to describe scalar particles!

So people were looking for another equation $\rightarrow$ Dirac Equation

DIRAC EQUATION

He WANTED equation with 1^{TH} derivatives terms

$$\boxed{\hat{E}\psi = (\vec{\alpha}\cdot\hat{\vec{p}} + \beta m)\psi} \qquad \text{Dirac Equation}$$

The solution of this equation must satisfy $E^2 = p^2c^2 + m^2c^4$

$\Rightarrow$ this way you can derive solution for the parameters $\vec{\alpha}$ and $\vec{\beta}$

Dirac Algebra

$$\begin{cases} \alpha_x^2 = \alpha_y^2 = \alpha_z^2 = \beta = I \\ \alpha_j \beta + \beta \alpha_j = 0 \\ \alpha_j \alpha_k + \alpha_k \alpha_j = 0 \quad (j \neq k) \end{cases}$$

let's write an explicit form for the $\vec{\alpha}$ and $\vec{\beta}$ parameter : Representation

One conventional is Dirac - Pauli :

Dimension?

- α and β are 4x4 complex equation

$$\beta = \begin{pmatrix} \mathbb{1}_2 & 0 \\ 0 & -\mathbb{1}_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$\alpha_i = \begin{pmatrix} 0 & \sigma_i \\ \sigma_i & 0 \end{pmatrix} \rightarrow \sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

The consequence is that, if $\vec{\alpha}$ and $\vec{\beta}$ are matrix, ψ is a 4-Dim vector

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix}$$

Dirac - Spinor

Covariant Form of the Dirac Equation

$$\hat{E}\psi = (\vec{\alpha} \cdot \hat{\vec{p}} + \beta m)\psi$$

$$i \frac{\partial}{\partial t}\psi = (-i\vec{\alpha}\vec{\nabla} + \beta m)\psi$$

$$i \frac{\partial}{\partial t}\psi = \left[-i\left(\alpha_x \frac{\partial}{\partial x} + \alpha_y \frac{\partial}{\partial y} + \alpha_z \frac{\partial}{\partial z}\right) + \beta m\right]\psi$$

we introduce Dirac Matrices as

$$\gamma^0 = \beta \qquad \gamma^1 = \beta\alpha_x \qquad \gamma^2 = \beta\alpha_y \qquad \gamma^3 = \beta\alpha_z$$

So multiplying by β from Right, and using $\beta^2 = \mathbb{1}_4$

$$i\beta \frac{\partial \psi}{\partial t} = (-i\beta \vec{\alpha}\vec{\nabla} + m)\psi$$

$$i\gamma^0 \frac{\partial \psi}{\partial t} = (-i\vec{\gamma}\vec{\nabla} + m)\psi \qquad \vec{\gamma} = \begin{pmatrix} \gamma_1 \\ \gamma_2 \\ \gamma_3 \end{pmatrix}$$

We introduce $\gamma^\mu = (\gamma_0, \gamma_1, \gamma_2, \gamma_3)$ ⚠ not a 4-vector

$$\partial_\mu = \left(\frac{\partial}{\partial t}, \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$$ → Definition of covariant 4-derivative

$\Rightarrow (i\gamma^\mu \partial_\mu - m)\psi = 0$ → · mc^2 is covariant
· ∂_μ is covariant

↳ Einstein convention

$\Rightarrow \boxed{(i\hbar\gamma^\mu\partial_\mu - mc)\psi = 0}$ Dirac Equation Covariant Form

Super Compact Form with Feynmann "Slash" notation

$$\not{A} = \gamma^\mu A_\mu \qquad \text{hence} \qquad \underline{(i\not{\partial} - m)\psi = 0}$$

Hiddens Behind:

① ψ is a spinor

② m is multiplied by $\mathbb{1}_4$

Remarks

- $\psi^+\gamma^0\psi = (\cdot\ \cdot\ \cdot\ \cdot)\begin{pmatrix} \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{pmatrix}\begin{pmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \end{pmatrix}$ = scalar $(1\times4)(4\times4)(4\times1) = 1\times1$

- $\psi^+\gamma^\mu\psi$ = vector $\mu = 0,1,2,3$

- We introduce $\bar{\psi} = \psi^+\gamma_0 = (\psi^*)^t\gamma^0$

  so:

  $\bar{\psi}\psi$ scalar

- We introduce $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 0 & \mathbb{1}_2 \\ \mathbb{1}_2 & 0 \end{pmatrix}$ In Pauli-Dirac Representation

Solutions

- The solution indeed a natural description of spin - half particle
- The probability density are >0 $\quad \rho = \psi^* \psi > 0$
- Some solutions have a negative energy $E<0$

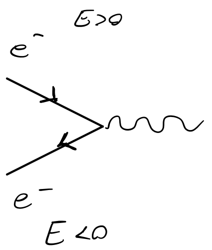
↳ Feynmann - Stuckelberg interpretation

if $E>0$ $\quad \longrightarrow$ $\quad \xrightarrow{\hspace{2cm}} t$ | PARTICLE

if $E<0$ $\quad \longleftarrow$ $\quad \xrightarrow{\hspace{2cm}} t$ propagates backward in time | ANTIPARTICLE

Feynmann Diagrams

- $e^- e^+$ annihilation



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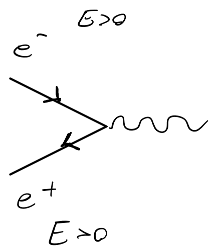
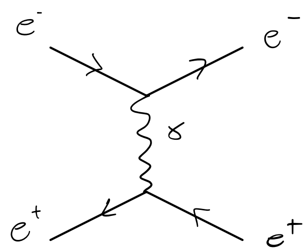


- $e^- e^+$ scattering



Spinors for particle and anti-particle for FREE SPIN-HALF

2) Particle $\qquad \psi_i = U_i e^{i(\vec{p}\cdot\vec{x} - Et)}$

$$U_1(P) = \sqrt{E+m} \begin{pmatrix} 1 \\ 0 \\ \frac{p_z}{E+m} \\ \frac{p_x + i p_y}{E+m} \end{pmatrix} \qquad \text{SPIN UP} \quad \uparrow$$

$$U_1(P) = \sqrt{E+m} \begin{pmatrix} 1 \\ 0 \\ \frac{p_x - i p_y}{E+m} \\ \frac{- p_z}{E+m} \end{pmatrix} \qquad \text{SPIN DOWN} \quad \downarrow$$

b) Antiparticle $\quad \psi_i = v_i \, e^{-i(\vec{p}\cdot\vec{x} - Et)}$

$$v_1(p) = \sqrt{E+m} \begin{pmatrix} \frac{p_x - i p_y}{E+m} \\ \frac{-p_z}{E+m} \\ 1 \\ 0 \end{pmatrix}$$

SPIN UP ↑

$$v_1(p) = \sqrt{E+m} \begin{pmatrix} \frac{p_x + i p_y}{E+m} \\ \frac{+p_z}{E+m} \\ 1 \\ 0 \end{pmatrix}$$

SPIN DOWN ↓

Compact Form:

PARTICLE $\Vert$ $u_{(s)} = \sqrt{E+m} \begin{pmatrix} \phi^{(s)} \\ \frac{\vec{\sigma}\cdot\vec{p}}{E+m} \cdot \phi^{(s)} \end{pmatrix}$

spin

$$\phi^{(1)} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \phi^{(2)} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

ANTI-PAR $\Vert$ $v_{(s)} = \sqrt{E+m} \begin{pmatrix} \frac{\vec{\sigma}\cdot\vec{p}}{E+m} \chi^{(s)} \\ \chi^{(s)} \end{pmatrix}$

$$\chi^{(1)} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad \chi^{(2)} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

with $\quad \vec{\sigma}\cdot\vec{p} = \sigma_1 \cdot p_x + \sigma_2 \cdot p_y + \sigma_3 \cdot p_z$

Completeness Relation

For particle $\quad \displaystyle\sum_{s=1}^{2} u_{(s)}(p)\,\bar{u}_{(s)}(p) = u_{(1)}(p)\,\bar{u}_{(1)}(p) + u_{(2)}(p)\,\bar{u}_{(2)}(p)$

$u_s(p)$

$$\bar{u}_{(s)}(p) = u_s^{+}\gamma_0 = \sqrt{E+m}\left(\phi_s^T - \phi^T \frac{\vec{\sigma}\cdot\vec{p}}{E+m} \right)$$

Hints for the calculation

- $(\vec{\sigma} \cdot \vec{p}) = \vec{p}^{\,2} = (E+m)(E-m)$
- $\sum_{s=0}^{1} \phi_{(s)} \phi_{(s)}^{T} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

- We find

$$\sum_{s=1}^{2} u_s \bar{u}_s = \left(\gamma^{\mu} p_{\mu} + m \mathbb{1}_4\right) = \not{p} + m$$

For antiparticle

$$\sum_{s=1}^{2} v_s \bar{v}_s = \not{p} - m$$

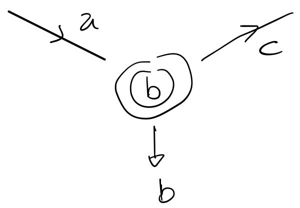
IV Interaction and Field

1) Introduction

Virtual Particle : γ^* virtual photon $\rightarrow$ The mass of this virtual γ is not 0

2) Interaction between particle $\quad a+b \rightarrow c+d$

a) Classical view (no QFT, no GR)



a interacts with the classical field of particle b $\rightarrow$ potential $V(\vec{R})$

described by KG equation

$$\vec{\nabla}^2 V(\vec{R}) - \frac{1}{\vec{R}_0^2} V(\vec{R}) = \delta(\vec{R})$$

• R_0 : tipical lenght of the interaction

EM : • ∞ - distance interaction $\quad \vec{\nabla}^2 V = \delta(\vec{R}) \rightarrow V(R) \propto \frac{1}{4\pi R}$

Strong Interaction :

$$V(\vec{R}) \propto \frac{e^{-R/R_0}}{4\pi R}$$

• Yukawa - Potential

• $R_0 = \frac{\hbar}{mc}$

$\hookrightarrow$ Mass of the particle that is exchanged : PIONS

Interaction Between Particles

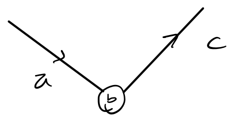
a) Classical view

KG $\longrightarrow$ EM potential

$\xrightarrow[\text{short distance}]{}$ $V(r) \propto \dfrac{e^{-mR}}{4\pi R}$ but introducing g_b : $V(r) = g_b \dfrac{e^{-mR}}{4\pi R}$

g_b : coupling constant of particle a with potential $V(R)$

Reminder scatter



g_a : coupling const associate with par. a

Scattering amplitude :

$$\mathcal{M} = g_a \int V(\vec{R})\, e^{i\vec{q}\cdot\vec{R}}\, d\vec{R}$$

$\vec{q}$: transfered momentum

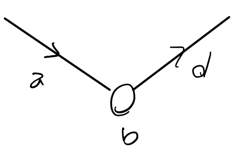
· The calculation gives $\mathcal{M} = \dfrac{g_a g_b}{|\vec{q}|^2 + m^2}$ $\quad m \rightarrow$ Mass of the transfered particle

Problem: • it fails for relativistic particle
• Too simplistic view of interactions between particle

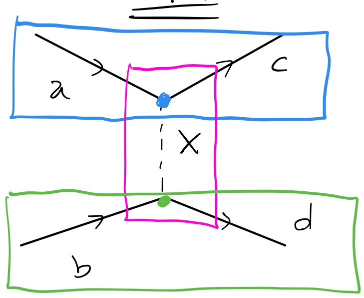
Moving to a more modern tool...

b) Interaction by particle exchange

Classical



QFT



X: virtual particle that is excanged

We introduce q: 4-momentum of the exchanged particle

$$q : P_a - P_c = P_b - P_d$$

$q^2 = E_X^2 - \vec{p}_X^{\,2} \neq m_X$ Virtual Particle $\sim$ is not on shell

Suppose that X is spin-0 (scalar) the matrix element of this process

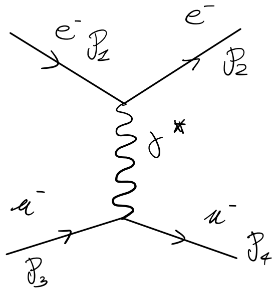
$$\mathcal{M}_{fi} = \frac{g_a g_b}{q^2 - m_X^2}$$ $\sim$ Similar to Classic one, but slightly different

More generally the matrix element for a scalar particle is composed of 3 terms:

$$\mathcal{M}_{fi} = \underbrace{\langle \psi_c | \hat{V} | \psi_a \rangle}_{1^\circ \text{ vertex}} \cdot \underbrace{\frac{1}{q^2 - m_X^2}}_{\text{Propagator}} \cdot \underbrace{\langle \psi_d | \hat{V} | \psi_b \rangle}_{2^\circ \text{ vertex}}$$

3) QED examples of Feynmann Rules

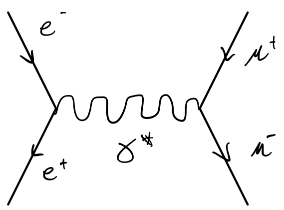
a) electron scatter



$$-i\mathcal{M} = \underbrace{\left[\bar{u}(p_3)\, ie\gamma^\mu u(p_1)\right]}_{1^\circ \text{ vertex}} \cdot \underbrace{\frac{-i g_{\mu\nu}}{q^2}}_{\text{Propagator}} \underbrace{\left[\bar{u}(p_4)\, ie\gamma^\nu u(p_2)\right]}_{2^\circ \text{ vertex}}$$

t-channel process
$$q_{\gamma^*}^2 = E^2 - \vec{p}^{\,2} < 0 \,!$$

b) s-channel $e^- e^+ \to \mu^- \mu^+$



$$-i\mathcal{M} = \underbrace{\left[\bar{v}(p_2)\, ie\gamma^\mu u(p_1)\right]}_{1^\circ \text{ vertex}} \cdot \underbrace{\frac{-i g_{\mu\nu}}{q^2}}_{\text{Propagator}} \cdot \underbrace{\left[\bar{u}(p_3)\, ie\gamma^\nu v(p_4)\right]}_{2^\circ \text{ vertex}}$$

S-channel process
$$q_{\gamma^*}^2 = E^2 - \vec{p}^{\,2} > 0 \,!$$

4) Interaction strenght

1. ELECTROMAGNETIC INTERACTION

We saw $\mathcal{M} \propto e^2$

$$\sigma \propto |\mathcal{M}|^2 \propto e^4$$

Coupling costant α

$\alpha = \dfrac{e^2}{4\pi\hbar c}$ For low energy process $\alpha \simeq \dfrac{1}{137}$

2. STRONG PROCESS

1) $K^- + p \longrightarrow \Sigma^0(1385) \longrightarrow \Lambda + \pi^0$ (STRONG because π^0, but we don't know)

2) $K^+ + p \longrightarrow \Sigma^0(1192) \longrightarrow \Lambda + \gamma$ (ELECTROMAGNETIC because of γ)

1) STRONG $\tau_1 \simeq 10^{-23}$ s

2) EM process $\tau_2 \simeq 10^{-11}$ s

We can calculate the ratio of the lifetime

$$\frac{\tau_2}{\tau_1} \propto \frac{\Gamma_1}{\Gamma_2} \propto \frac{\alpha_s^2}{\alpha^2}$$ with α_s: STRONG COUPLING CONSTANT

$$\boxed{\alpha_s = \frac{g_s}{4\pi}}$$

becaus $\Gamma = \dfrac{\hbar}{\tau}$ and $\Gamma \propto |\mathcal{M}|^2 \simeq e^4 \simeq \alpha^2$

So $= \left(\dfrac{10^{-14}}{10^{-23}}\right)^{1/2} \simeq 100 \propto \left(\dfrac{\alpha_s^2}{\alpha^2}\right)^{1/2} \longrightarrow \alpha_s \simeq 100\alpha$

3. WEAK INTERACTION

The mediator of weak interaction is the W boson

$\Rightarrow$ massive boson propagator

Let's consider a decay

$$\Sigma^- \to n + \pi^- \quad \text{weak process} \quad \tau \simeq 10^{-10}\, s$$

$$\Sigma^0 \to \Lambda + \gamma \quad \text{EM proc} \quad \tau \simeq 10^{-19}\, s$$

Ratio of lifetimes $\quad \dfrac{\tau_{EM}}{\tau_{weak}} \propto \left(\dfrac{\alpha_W^2}{(q^2 - m_W)^2} \cdot \dfrac{1}{\alpha^2} \right)$

$\rightsquigarrow$ For low energy process $q^2 \ll m_W^2$

$$\left(\frac{\tau_{EM}}{\tau_{weak}} \right)^{1/2} \propto \frac{\alpha_W^2}{m_W^2\, \alpha} \simeq 10^{-5}$$

For High energy process $(q^2 - m_W^2)^2 \simeq q^2$ so the weak int. is not that weak

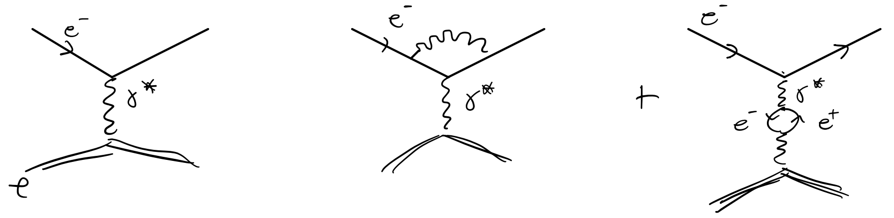
α_W:

$$G_F = \frac{\sqrt{2}}{8} \frac{g_W^2}{m_W^2} = 1.16\ 10^{-5}\, GeV^{-2}$$

5) High order effects

Introduction

1° | Lamb's shift : observation of unknown spectral line in the hidrogen atom
$\lambda = 28,4$ cm

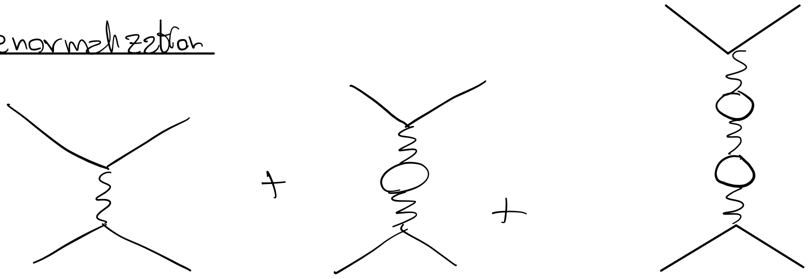


2° | Magnetic Moment

$$\vec{\mu} = g \frac{Q}{2m} \vec{S} \quad \text{with } g \text{ : giromagnetic ratio}$$

Should be 2 but if you consider higher effects, $g \neq 2$

Renormalization

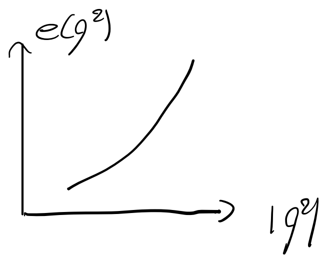


Cross Section + ∞ + ∞

Problem!

$\Longrightarrow$ Solved by define the electric charge $\underline{e_0 = e(q^2)}$ Variable

Indeed was measured that e change depending on the $|q|^2$



Chapter II - Introduction to Strong Interaction

1) Quark Model

1.1 Quarks and Colours

$$\begin{pmatrix} u \\ d \end{pmatrix} \Big| \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix}$$

MATTER DECAY

Top quark is the only one decay 100% $t \to Wb$
173 GeV 80 GeV

Quarks glued togheter can produce Hadrons

HADRONS

- Mesons $|q\bar{q}'\rangle$ $q = u, c, d, b, s$ NOT t
- Baryons $|qq'q''\rangle$ $q = u, c, d, b, s$ NOT t
  AntiBaryon $|\bar{q}\bar{q}'\bar{q}''\rangle$

Example:

Neutron $|uud\rangle$
Proton $|udd\rangle$

Whit the discover of $\Delta^{++} \to \pi^+ + p$ we know that $\Delta^{++} = |u\uparrow u\uparrow u\uparrow\rangle$

$\to$ Fermi Principle NOT ALLOWED $\rightsquigarrow$ 3 fermions with same quantum numbers

Something missing, so existence of $\boxed{\text{COLOUR}}$

$$\Delta^{++} = \{|RGB\rangle + |BRG\rangle + |GBR\rangle\}$$

Going back to the mesons with this new colour

Meson $|q\bar{q}'\rangle \Rightarrow$ Color - ANTIcolor state

Following the way we write color, we know that the COLOR COMBINATION has to be NEUTRAL

MESONS: Color - anticolor states

BARYONS: States that have all RGB colors

So the TRUE WAVE FUNCTION is a combination of the color states

Mesons:
$$\frac{1}{\sqrt{3}}\left(|R\bar{R}\rangle + |G\bar{G}\rangle + |B\bar{B}\rangle\right)$$

Baryon:
$$\frac{1}{\sqrt{6}}\left(|RGB\rangle - |GRB\rangle + |GBR\rangle + \dots\right)$$

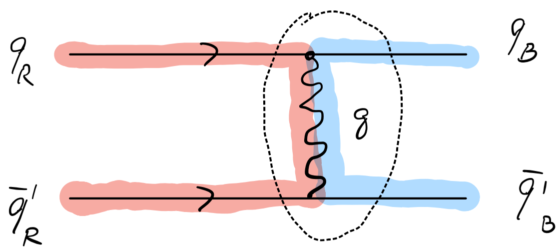
1 permutation means $(-1)\cdot|\psi\rangle$

Gluons

Are GAUGE boson mediators of strong interaction

Mass is 0

Are made of 2 colors



Double Color

⤳ In quark-gluon interaction they change the color of the quark

\# gluons: 3 colors × 3 anticolors − 1 colorless STATE = 8

$(R\bar{R} + G\bar{G} + B\bar{B})$

1.2 Symmetries in Quantum Mechanics

$$\psi \to \psi' = U\psi$$

where U is a symmetry operation

This operation change the state but leave physical predictions unchanged

$\langle\psi|\psi\rangle = \langle\psi'|\psi'\rangle = \langle\psi|U^\dagger U|\psi\rangle \to U^\dagger U \stackrel{!}{=} \mathbb{1}$ so U is unitary

Let's consider an infinitesimal symmetry operation

$$U = \mathbb{1} + i\varepsilon G$$

↳ Generator of Transformation

↳ Infinitesimal Transformation Parameter

Requirement: $U^\dagger U = \mathbb{1}$ ⇝ We find out $\underbrace{G^\dagger = G}_{}$

G is Hermitian

Finite Transformation θ

$$U(\theta) = \lim_{n\to\infty}\left(\mathbb{1} + i\frac{\theta}{n}G\right)^n = \exp(i\theta G)$$

Generalization to several parameters of the transformation $\vec{\theta}$

$$U(\vec{\theta}) = \exp(i\,\vec{\theta}\,\vec{G})$$

1.3 Isospin and SU(2)

Proton and neutron behave in a similar way for strong interaction

⇒ Proton and Neutron are (almost) the same

Isospin reflects that symmetry (2 state that behave in the same way)

⇝ 2 STATE SIMMETRY

Isospin State $|p\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $|n\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

But proton and neutron are made of u and d quarks, so we can propagate this to the quark

$|u\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $|d\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

What is the underlying symmetry?

- Must satisfy $U^\dagger U = \mathbb{1}$:

CONSEQUENCE: $\det(U) = e^{i\beta}$

Therefore the value of the determinant is fixed $\underbrace{|\det(U)|^2 = 1}$

We call it Special Symmetry SU(2)

2? → 2 state symmetry

We show that

$$U(\vec{\theta}) = e^{i\vec{\theta}\vec{T}}$$

$$= e^{i(\theta_1\sigma_1 + \theta_2\sigma_2 + \theta_3\sigma_3)}$$

- $\vec{\theta}$ of dimension 3
- $\vec{T} = \frac{1}{2}\vec{\sigma}$ the generator

:= vector of Pauli matrices $\sigma = (\sigma_1, \sigma_2, \sigma_3)$

1.4 SU(3) group

COLOR SYMMETRY: $SU(3)_c \to$ QCD is unchanged in the permutation of colors (R, G, B)

"Exact Symmetry" $\Rightarrow$ Red is not more Red than Blue!

FLAVOR SYMMETRY: $SU(3)_f \to u, d, s$ have $m_u \sim m_d \sim m_s$

"Broken Symmetry" $\Rightarrow$ In realita $m_s \gg m_u \sim m_d$

WHAT IS U?

Requirement: • $U^\dagger U = \mathbb{1}$

• $|\det(U)|^2 = 1$

we showed $U(\vec{\theta}) = e^{\frac{i}{2}\vec{\theta}\vec{\lambda}} = e^{\frac{i}{2}\sum_{i=1}^{8}\theta_i \lambda_i}$

• $\vec{\theta}$: vector of 8 parameter

• $\vec{\lambda}$: Generator, vector of Gell Mann Matrices

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_2 = \begin{pmatrix} & -i & \\ i & & \\ & & \end{pmatrix} \quad \lambda_3 = \begin{pmatrix} 1 & & \\ & -1 & \\ & & \end{pmatrix}$$

$$\lambda_4 = \begin{pmatrix} & & 1 \\ & & \\ 1 & & \end{pmatrix} \quad \lambda_5 = \begin{pmatrix} & & -i \\ & & \\ i & & \end{pmatrix} \quad \lambda_6 = \begin{pmatrix} & & \\ & & 1 \\ & 1 & \end{pmatrix}$$

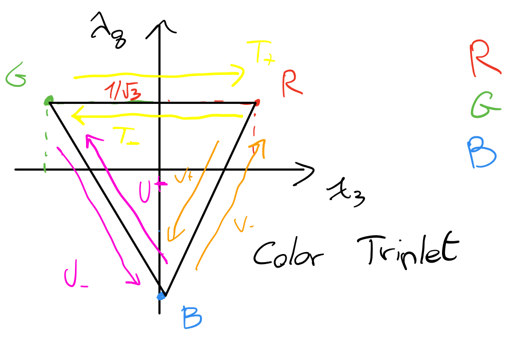
$$\lambda_7 = \begin{pmatrix} & & \\ & & -i \\ & i & \end{pmatrix} \quad \lambda_8 = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & & \\ & 1 & \\ & & -2 \end{pmatrix}$$

We can write $R = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad G = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad B = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

Illustration

Represent this 3 states in the space of

→ Calculate heigen value of R, G, B in respect λ_3 and λ_8 and draw heigenstate



$T_\pm = \frac{1}{2}(\lambda_1 \pm i\lambda_2)$ $\leadsto$ $T_- R = G$ "Isospin Ladder Operators"

$V_\pm = \frac{1}{2}(\lambda_4 \pm i\lambda_5)$

$U_\pm = \frac{1}{2}(\lambda_6 \pm i\lambda_7)$

1.5) $SU(3)_f$

u $m(u) \sim 2$ MeV
d $m(d) \sim 6$ MeV
s $m(s) \sim 100$ MeV

Not an exact symmetry

- It's used for classification of mesons and baryon → you can predict new particle
- Also you can predict the mass of the hadron

Gave rise on QUARK-MODEL

We can build triplets as we did in the last symmetry

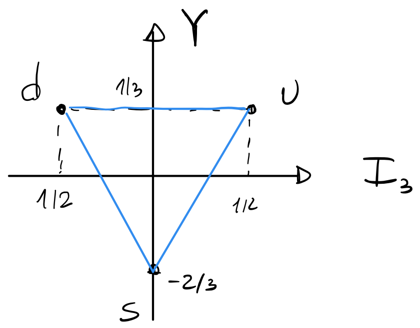
Hipercharge $Y = B + S$

Barion : B $+\frac{1}{3}$ quarks , $-\frac{1}{3}$ antiquarks

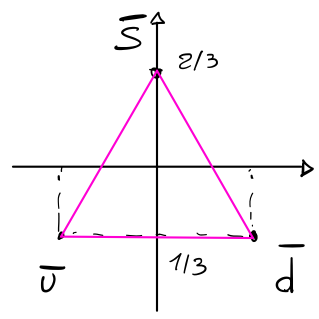
→ u, d have isospin
s no!

Strange: S -1 s quark , $+1$ $\bar{s}$ quark

Visualization

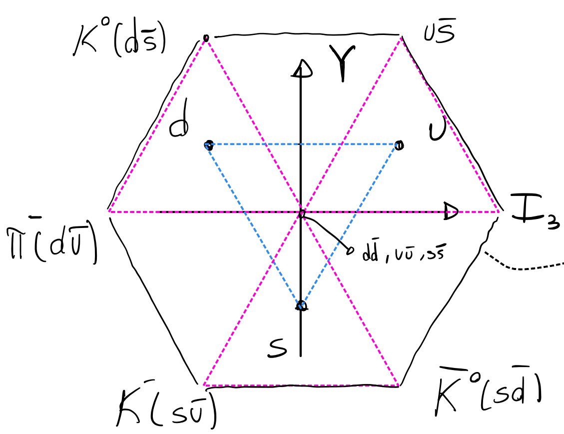


Symbol 3



Symbol $\bar{3}$

Building Mesons $(q\bar{q})$



Center:

$\pi^0 \quad |u\bar{u} - d\bar{d}\rangle \frac{1}{\sqrt{2}}$

$\eta \quad |u\bar{u} + d\bar{d} - 2s\bar{s}\rangle \frac{1}{\sqrt{6}}$

$\eta' \quad |u\bar{u} + d\bar{d} + s\bar{s}\rangle \frac{1}{\sqrt{3}}$ SINGLET

$$\underbrace{\qquad\qquad\qquad}_{3 \otimes \bar{3} = 8 \oplus 1}$$

Building Lightest Baryon

$$3 \otimes 3 \otimes 3 = 10 \oplus 8 \oplus 8' \oplus 1$$

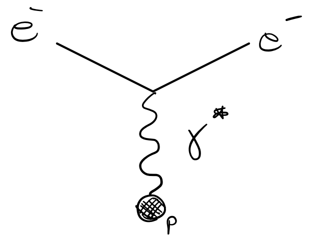
Remark

Thake the 4th haviest quark you can build $SU(4)_f$
⤳ good for classification but not for masses

We can add the 5^{th} and build $SU(5)$ BUT we cannot make $SU(6)$ because the t is too heavy and we are not able to build stable particles because he decays fregnoltilly

2. Deep Inelastic Scattering

2.1) Probing the structure of the proton



Probe the proton at different energy

$$\lambda \sim \frac{1}{\sqrt{-q^2}} \rightsquigarrow \text{Resolution Wave lenght}$$

⤳ It's not negative: at relativistic $q^2 < 0 \gg Q = -q^2 > 0$

if $\lambda \gg R_p$ Rutherford

if $\lambda \sim R_p$: Sensitive to charge distribution in the proton

if $\lambda \ll R_p$: Sensitive to quark-gluons structure

2.2 Form Factor

When $\lambda \sim R_p$ you start to see charge structure of the p

→ For very low energy Rutherford (non-relativistic)
Mott (relativistic)

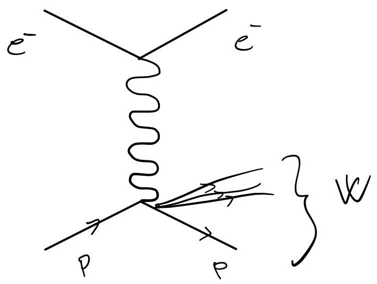
→ For heigh energy the differential cross section change with the angle

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \times \underbrace{|F(q)|^2}_{\text{Form Factor}}$$

For the photon:

- Rosenbluth Formula with 2 Factor: charge momentum, magnetic momentum

2.3) Inelastic Scattering at low Q^2



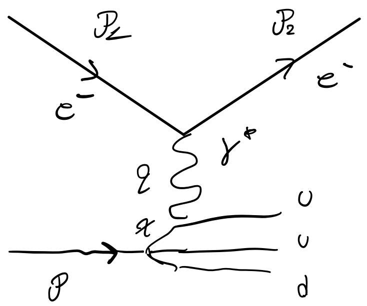
Production of excited state or other particle + the proton at energy 100 MeV – 10 GeV

$\Longrightarrow$ Proton is not point-like particle

We introduce the "structure functions" to describe inelastic scattering

2.4) Deep Inelastic scattering – Parton model

At very high energies the proton is broken and the photon interacts with its constituents (partons)



The quarks in the proton are almost free particles

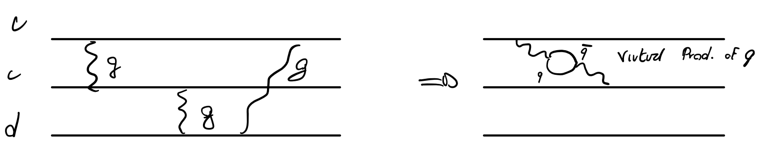
$\rightsquigarrow$ The interaction is so that you need to consider the momentum of each quark

$$P_q = x\, P_{proton} \quad \longrightarrow \quad x = \frac{Q^2}{2 P_2 \cdot q}$$

For each quark flavour the probability distribution of x has been measured $\Longrightarrow$ PDF "Parton Distribution Function"

$\Longrightarrow$ This picture is not totally TRUE

Quarks interacts with each-other by exchanging gluons



$\longrightarrow$ u, d, s, c, b

"Sea Quarks"

$\Longrightarrow$ we have PDF for the 5 quarks in the proton

2.5 Hadronic cross-section - Factorization theorem

LHC pp collision

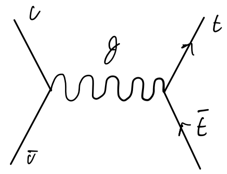
$$pp \to t\bar{t}$$

$$\sigma(pp \to t\bar{t}) = \sum_{ij} \int \underbrace{f_i(x_i) f_j(x_j)}_{\text{PDF}_i \ \text{PDF}_j} \, dx_i \, dx_j \, \underbrace{\hat{\sigma}(ij \to t\bar{t})}_{\text{HARD PROCESS QFT}}$$

EXPERIMENTALLY

$dx_i \to$ Fraction of momentum of single proton

For this process



(if you have only electron it's only the Hard Process term)

Chapter III - Weak Interaction

1) Experimental evidence

$\Delta^{++} \to p\,\pi^{+}$	$\sim 10^{-3}\,s$	ST	
$\Sigma^{0} \to \Lambda\,\gamma$	$\sim 6\cdot 10^{-20}\,s$	EM	
$\Sigma^{-} \to n\,\pi^{-}$	$\sim 10^{-10}\,s$	WI	Flavor change
$S=1$ $\quad$ $S=0$			

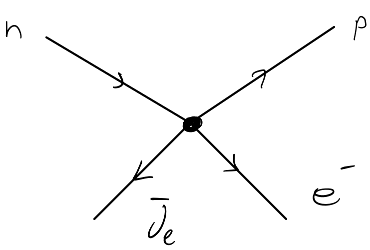
$\pi^{-} \to \mu^{-}\bar{\nu}_{\mu}$ $\quad \sim 10^{-8}\,s$

$n \to p\,e^{-}\bar{\nu}_{e}$ $\quad \sim 15\,min$

$\beta^{\pm}$ decay Radioactivity

Solar nuclear Flavor

Early '30 Enrico Fermi developed a model for weak interaction in analogy with QED



"contact" interaction

$$\to \boxed{\mathcal{M} = G_F\left(\bar{\psi}_p \Gamma \psi_n\right)\left(\bar{\psi}_e \Gamma \psi_{\bar{\nu}_e}\right)}$$

with Γ : some matrix describe the weak process

1956 Weak interaction violates Parity $\Rightarrow$ V - A structure

$\uparrow$ will be modified

2 Parity

Spatial coordinate inversion $\vec{R} \to -\vec{R}$, in QM can be associated to Parity operator $\hat{P}$

$$\Psi(\vec{R},t) \xrightarrow{\hat{P}} \hat{P}\,\Psi(\vec{R},t) = \Psi(-\vec{R},t)$$

So $\hat{P}\hat{P} = \mathbb{1}$

↳ $\hat{P}$ operator has $P = \pm 1$ eigenvalues

If an interaction is invariant under $\hat{P}$ then $\hat{P}\hat{P} = \mathbb{1}$
$\Rightarrow \hat{P}^{+} = \hat{P}$ Hermitian

Intrinsic Parity of Fermions

Dirac Equation $\quad (i\gamma^{\mu}\partial_{\mu} - m)\,\Psi(\vec{R},t) = 0$

~~Applying Parity Transformation gives:~~

We can write $\gamma^{\mu}\partial_{\mu} = \gamma^{0}\frac{\partial}{\partial t} + \vec{\gamma}\vec{\nabla}$

So:

$$\left(i\gamma^{0}\frac{\partial}{\partial t} - i\vec{\gamma}\vec{\nabla} - m\right)\Psi(-\vec{R},t) = 0$$

We multiply left by γ^{0} and use $\underbrace{\gamma^{0}\gamma^{i} + \gamma^{i}\gamma^{0} = 0}_{\gamma^{0}\vec{\gamma} = -\vec{\gamma}\gamma^{0}}$ with $i = 1,2,3$

$$= \left(i\gamma^{0}\gamma^{0}\frac{\partial}{\partial t} + i\vec{\gamma}\gamma^{0}\vec{\nabla} - m\gamma^{0}\right)\Psi(-\vec{R},t) = 0$$

$$= \left(i\frac{\partial}{\partial t} + \vec{\gamma}\vec{\nabla} - m\right)\gamma^{0}\Psi(-\vec{R},t) = 0 \qquad \gamma^{0}\text{ Dirac Eqution}$$

so $\boxed{\gamma^{0}\Psi(-\vec{R},t) = \Psi(\vec{R},t)}$ Parity is the same as multiply by γ^{0}

Parity of fermions

$$u^{(1)} = \sqrt{E+M}\begin{pmatrix} 1 \\ 0 \\ P_3/(E+m) \\ P_x + iP_y/(E+m) \end{pmatrix} \longrightarrow \hat{P}u^{(1)} = \sqrt{E+M}\begin{pmatrix} 1 \\ 0 \\ -P_3/(E+m) \\ -P_x - iP_y/(E+m) \end{pmatrix}$$

It's the same multiplying γ^{0}?

Reminder: $\gamma^{0} = \begin{pmatrix} \mathbb{1}_2 & 0 \\ 0 & -\mathbb{1}_2 \end{pmatrix}$

$$= +\gamma^0 u^{(1)}$$ $\Longrightarrow$ So intrinsic parity of Fermion is $+1$

With same result with $u^{(2)}$

Parity of antifermions

$$v^{(1)} = \sqrt{E+m}\begin{pmatrix} p_3/(E+m) \\ p_1+ip_2/(E+m) \\ 1 \\ 0 \end{pmatrix} \longrightarrow \hat{P} v^{(1)} = \sqrt{E+m}\begin{pmatrix} -p_3/(E+m) \\ -p_1-ip_2/(E+m) \\ 1 \\ 0 \end{pmatrix} = -\gamma^0 v^{(1)}$$

$\Longrightarrow$ So intrinsic parity of ANTIFERMION is -1

Intrinsic Parity of bosons

$$P(\gamma) = P(W) = P(Z) = .. = -1$$

Property of parity

Parity of a sistem of particles = Product of intrinsic parities of particles times spatial wave function

Example: Positronium e^+e^-

$$P(e^+e^-) = P(e^+)\cdot P(e^-)\cdot(-1)^L = (-1)^{L+1}$$

3 Charge Conjugation $\hat{C}$

- $\hat{C}$ reverse the spin of the charge (electric, colour) and MAGNETIC MOMENT
- $\hat{C}\hat{C} = \mathbb{1} \rightarrow C = \pm 1$ eigenstates
- $\hat{C}$ will change a particle into an antiparticle $\gg$ system of Partic-Antip are eigenstates
- EM is $\hat{C}$ invariant
  $\Rightarrow C_\gamma = -1$

Remark: Parity and charge are use to categorize meson J^{PC}

4. Time Reversal $\hat{T}$

- $\hat{T}\,\psi(\vec{R},t) = \psi(\vec{R},-t)$
- $\hat{T}^2 = \mathbb{1} \Rightarrow T = \pm 1$ eigenvalues

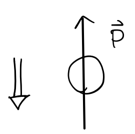
CPT theorem

All interactions described by a local Lorentz invariant gauge theory must be invariant under the combination of $\hat{P}, \hat{C}, \hat{T}$

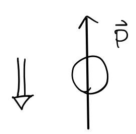
Consequence:
$$\hat{P} \Leftrightarrow \hat{C}\hat{T}$$
$$\hat{C} \Leftrightarrow \hat{P}\hat{T}$$
$$\hat{T} \Leftrightarrow \hat{C}\hat{P}$$

5) Helicity

- $h = \dfrac{\vec{S}\cdot\vec{P}}{\|\vec{S}\|\cdot\|\vec{P}\|}$ $\rightsquigarrow$ For spin 1/2 $\quad \vec{S} = \frac{1}{2}\vec{\sigma}$ and $h = \dfrac{\vec{\sigma}\cdot\vec{p}}{\|\vec{p}\|}$
- $h = \begin{cases} +1 & \text{RIGHT HANDED} \\ -1 & \text{LEFT HANDED} \end{cases}$



RIGHT HANDED



RIGHT HANDED

Helicity is relative for particles with $m \neq 0$
$\rightarrow$ depends on the REFERENCE FRAME

6) Weak Interaction and Parity

6.1. Parity and EM and strong interaction

Experimental Fact: Parity is conserved for EM and STRONG interaction

$\Longrightarrow$ Only parity-conserved process is allowed

Parity of Pion

$$\pi^+ = |u\bar{d}\rangle$$

$$P(\pi^+) = (+1)(-1)(-1)^L \qquad S=0,\ J=0 \Rightarrow L=0$$

$$\overset{!}{=} -1$$

Parity of ρ

$$\rho = |u\bar{d}\rangle$$

$$P(\rho) = (+1)(-1)(-1)^L \qquad S=1,\ J=1 \Rightarrow L=0$$

$$\overset{!}{=} -1$$

Exercise

a) $\rho^0 \to \pi^+ + \pi^-$

$$J^P = 1^- \to 0^- + 0^- \quad \Longrightarrow \quad P(\rho^0) = P(\pi^+)\,P(\pi^-)\,(-1)^L$$

$$\overset{!}{=} (-1)(-1)(-1) \qquad \text{ok}$$

b) $\eta \to \pi^+ + \pi^0$

$$0^- \to 0^- + 0^- \quad \Rightarrow \quad P(\eta) = P(\pi^+)\,P(\pi^-)\,(-1)^L \qquad L=0$$

$$-1 \overset{!}{\neq} (-1)(-1)$$

Not observed

6.2. The θ/τ puzzle

Early '50 observation of 2 decays

$$\tau \to \pi^+ + \pi^- + \pi^+$$

$$\theta \to \pi^+ + \pi^0$$

τ and θ have the same mass, same lifetime and same $S=0$, $L=0$

$$P(\tau) = (-1)(-1)(-1)^{L=0} = -1$$

$$P(\theta) = (-1)(-1)(-1)^{L=0} = +1$$

τ and θ are the same particle $\Rightarrow K^+$

Remark: Parity inversion properties of Physical Quantity

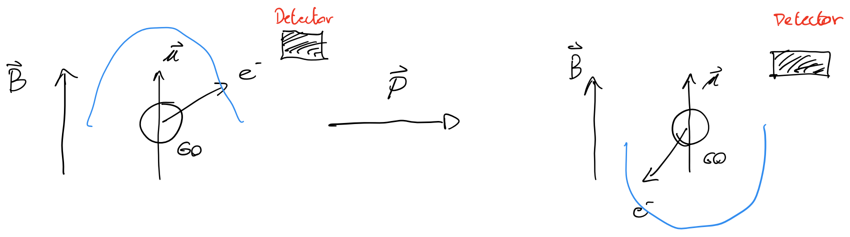
- SCALAR: unchanged under $\hat{P}$
- VECTORS $(\vec{R}, \vec{Q}, \dots)$: change sign $\vec{R} \to -\vec{R}$
- AXIAL VECTOR (cross product of 2 vectors $\vec{B} = \vec{\nabla} \times \vec{A}$
  $\vec{L} = \vec{R} \times \vec{P}$)
- PSEUDO SCALARS : scalar product of a vector and axial vector
  (ex: helicity $h = \frac{\vec{s} \cdot \vec{p}}{\|\vec{s}\| \cdot \|\vec{p}\|} \Rightarrow$ change sign $\hat{P}$)

C.W. experiment

$$^{60}Co \longrightarrow {}^{60}Ni^{**} + e^- + \bar{\nu}_e$$

$\underbrace{^{60}Co}$ Small Magnetic Moment $\vec{\mu}$

Aligned in a strong magnetic Field $\vec{B}$

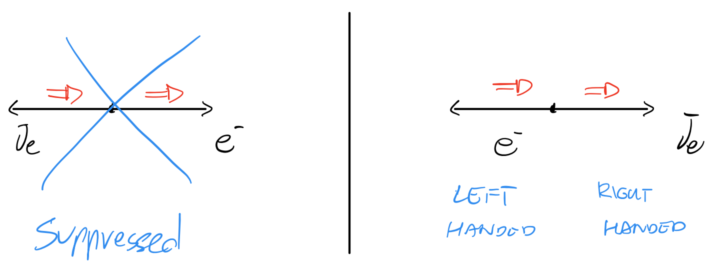


The experiment ⇒ ~20% more emitted backward ⇒ PARITY VIOLATE

Spin

^{60}Co $J=5$ ⇒

^{60}Ni $J=4$ ⇒



Conclusion: Weak Interaction applies with
- LEFT HANDED ANTIPARTICLE
- RIGHT HANDED PARTICLE

7) V−A structure of the weak interaction

7.1 Bilinear covariant combination

Structure of QED $\quad \bar{\psi}\gamma^\mu\psi$

Generic interaction $\quad \bar{\psi}\,\Gamma\,\psi$

↳ Matrix combining the γ matrices

In the SM there are only 5 possible combination / those combination are called bilinear covarb.

Type	Form	# components	spin
SCALAR	$\bar{\psi}$	1	0
PSEUDOSCALAR	$\bar{\psi}\gamma^5\psi$	1	0
VECTORS	$\bar{\psi}\gamma^\mu\psi$	4	1
AXIAL - VECTORS	$\bar{\psi}\gamma^\mu\gamma^5\psi$	4	1
TENSORS	$\bar{\psi}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)\psi$	6	2

<u>Definition</u> CHIRALITY STATES

- It's intrinsic property of Matter
- 2 states: LH - chirality

  RH - chirality

- Introduced by γ^5 matrix

$$\gamma^5 = \begin{pmatrix} 0 & \mathbb{1}_2 \\ \mathbb{1}_2 & 0 \end{pmatrix}$$

Pauli - Dirac

- Chirality states are eigenvector of γ^5
- For mass less particle, chirality = helicity

  For massive relativistic particle chirality $\simeq$ helicity

  For NON - RELATIVISTIC not so sure

- Chirality projection operator P_L, P_R

$$P_L = \frac{1}{2}(1-\gamma^5) \qquad P_R = \frac{1}{2}(1+\gamma^5)$$

<u>Properties:</u>

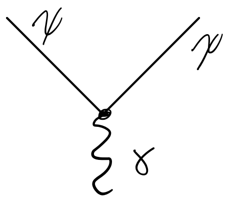
$$P_L + P_R = 1$$
$$P_L P_R = P_R P_L = 0$$
$$P_L^2 = P_L \quad \text{and} \quad P_R^2 = P_R$$

- Chiral components of the spinor

$$\psi = (P_R + P_L)\psi = P_R\psi + P_L\psi = \psi_R + \psi_L$$

7.3 Chiral structure of weak interaction

First let's look at the chiral structure of QED



$$\bar\psi\gamma^\mu\psi = (P_R+P_L)\bar\psi\gamma^\mu(P_R+P_L)\psi$$

$$\stackrel{!}{=} \cancel{\bar\psi P_R\gamma^\mu P_R\psi} + \bar\psi P_L\gamma^\mu P_R\psi + \bar\psi P_R\gamma^\mu P_L\psi + \cancel{\bar\psi P_L\gamma^\mu P_L\psi}$$

Reminder $\gamma^5\gamma^\mu = -\gamma^\mu\gamma^5$

$$\bar\psi P_R\gamma^\mu P_R\psi = \cancel{\bar\psi P_R P_L\gamma^\mu\psi}$$

$$\bar\psi P_L\gamma^\mu P_R\psi = \bar\psi P_L P_L\gamma^\mu\psi$$

For QED $\bar\psi\gamma^\mu\psi = \bar\psi P_L\gamma^\mu P_R\psi + \bar\psi P_R\gamma^\mu P_L\psi$

Since $P_L\psi = \psi_L$ $\qquad P_R = \psi_R$

$\bar\psi P_L = \bar\psi_R$ $\qquad \bar\psi P_R = \bar\psi_L$

Hence $\bar\psi\gamma^\mu\psi = \bar\psi_R\gamma^\mu\psi_R + \bar\psi_L\gamma^\mu\psi_L$ No VIOLATION

$\rightsquigarrow$ QED interact both with RH, LH

In weak interaction Parity is violated $\Rightarrow$ The chiral structure must be different

By playing with bilinear covariant form we find that the weak interaction has a V−A structure: (has been proved experimentally)

$$\bar\psi\gamma^\mu\underbrace{(1-\gamma^5)}_{2P_L}\psi = 2\bar\psi\gamma^\mu P_L\psi = 2\bar\psi\gamma^\mu P_L^2\psi \stackrel{!}{=} 2\bar\psi P_R\gamma^\mu P_L\psi \stackrel{!}{=} 2\bar\psi_R\gamma^\mu\psi_L$$

Only LH-chirality state can interact with Weak Force

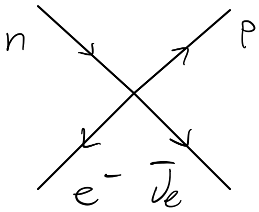
V-A $\Rightarrow$ Vector minus Axial Vector

Remark

V+A structure also violate parity

Twist experiment (2001-2007) : study decay 10^{10} muons $\Rightarrow$ proves that weak interaction is V-A

7.4 Return on Fermi Theory



1934 !

$$\mathcal{M} = G_F (\bar{\psi}_p \Gamma^\mu \psi_n)(\bar{\psi}_e \Gamma^\mu \psi_{\bar{\nu}_e})$$

1957

$$\Gamma^\mu = \gamma^\mu$$

$$\Gamma^\mu = \frac{1}{2}\gamma^\mu(1-\gamma^5)$$

$\Rightarrow$ low energy weak interaction

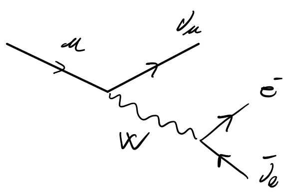
$$|q^2| \ll m_W^2$$

7.5 W boson propagator

- Weak interaction : excange of W boson

Propagator $\dfrac{-i g_{\mu\nu}}{q^2 - m_W^2}$

- Matrix element for weak decay $\mu^- \to e^- \bar{\nu}_e \nu_\mu$



- Matrix Element

$$\boxed{\mathcal{M} = \left[-\bar{\psi}\,\frac{g}{\sqrt{2}}\,\frac{1}{2}\gamma^\mu(1-\gamma^5)\psi_1\right] \cdot \frac{g_{\mu\nu}}{q^2 - m_W^2} \cdot \left[\bar{\psi}_f\,\frac{g}{\sqrt{2}}\,\frac{1}{2}\gamma^\mu(1-\gamma^5)\psi_2\right]}$$

At limit $|q^2| \ll m_W^2$

$$\mathcal{M} = \underbrace{\frac{g^2}{8 m_W^2}}_{\frac{G_F}{\sqrt{2}}} \left[\bar{\psi}_3 \gamma^\mu (1-\gamma^5) \psi_1 \right] \cdot \left[\bar{\psi}_4 \gamma^\mu (1-\gamma^5) \psi_2 \right]$$

$$\longrightarrow \quad \boxed{G_F = \frac{\sqrt{2}\, g^2}{8 m_W^2}}$$

Fermi Constant

GF can be measured

Indeed we have

$$\Gamma(\mu^- \to e^- \nu_\mu \bar{\nu}_e) = \frac{1}{\tau_\mu} = \frac{G_F^2 m_\mu^5}{192 \pi^3}$$

Precise measurements of τ_μ and m_μ gives

$$G_F = 1.16 \; 10^{-5} \; \mathrm{GeV}^{-2}$$

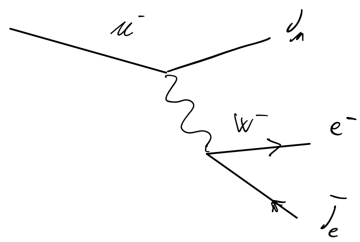
and so $\quad m_W = 80,385 \pm 0,015$ GeV

We can indeed measure the WEAK COUPLING CONSTANT $\quad \boxed{\alpha_W^2 = \frac{g^2}{4\pi} = \frac{8 m_W^2 G_F}{4\sqrt{2}\,\pi} \simeq \frac{1}{30}}$

8) CKM Matrix

Muon Decay

$G_F \propto g^2$



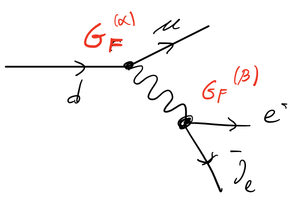
Similarly for tau decay

$$G_F^{(e)} = G_F^{(\mu)} = G_F^{(\tau)}$$

Leptonic Universality

Not true for weak interaction involving quarks:

Example β- decay



Experimentally $G_F^{(\alpha)}$ is 6% lower than $G_F^{(\beta)}$

WHY?

An even more visible process is K

$K^- \to \mu^- \bar{\nu}_\mu$

$\pi^- \to \mu^- \bar{\nu}_\mu$

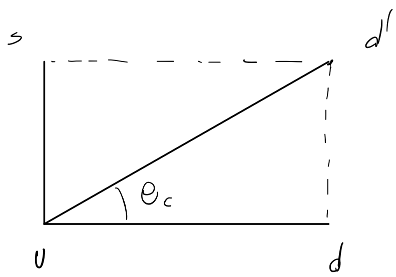
decay for K^- is 20 times smaller than π^- decay

$\rightsquigarrow$ There is something that change the power of weak interaction when quarks are involved

Nicola Cabibbo hypotesis (1963)

There is a difference : For weak interaction there is a distinction between WEAK EIGENSTATES and MASS EIGENSTATES

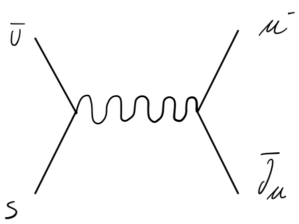
Idea: Instead of having $u - d$ coupling we have $u - d'$ where d' is the weak eigenstate



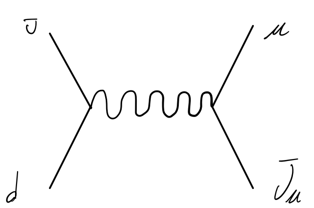
$\theta_c :=$ cabibbo angle

$\rightsquigarrow \; g_{ud} = g \cos\theta_c$

$\rightsquigarrow \; g_{us} = g \cos\theta$



$|\mathcal{M}_K|^2 \propto g^2 \sin^2\theta_c$



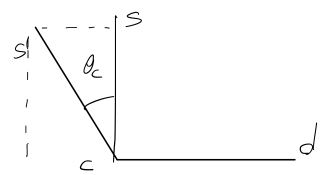
$|\mathcal{M}_\pi|^2 \propto g^4 \cos^2\theta_c$

$$\frac{|\mathcal{M}_K|^2}{|\mathcal{M}_\pi|^2} \propto \tan^2\theta_c = \underline{0.05}$$

$\theta_c \simeq 13°$

$\rightsquigarrow$ This explain the suppressed factor difference between K and π decays

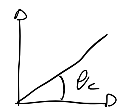
Similarly when we introduce the 4th quark $c \leftrightarrow s'$



$$g_{cd} = -g \sin\theta_c$$
$$g_{cs} = g \cos\theta_c$$

More generally weak eigenstates d' and s' are related to the mass eigenstates d and s by a matrix

$$\boxed{\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}}$$



GIM Mechanism (Glashow, Iliopoulos, Maiani) 1970

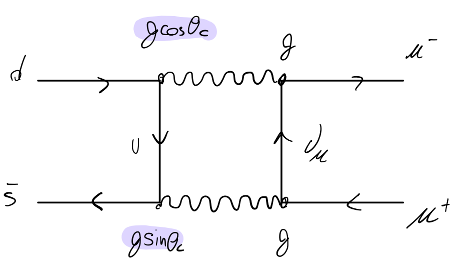
Before the c quark discovery

Taon decay EXPECTED but experimentally not observed

$$K_L^0 \to \mu^+\mu^-$$

Indeed $BR(K^0 \to \mu^+\mu^-) = 6.89 \cdot 10^{-9}$

Explanation

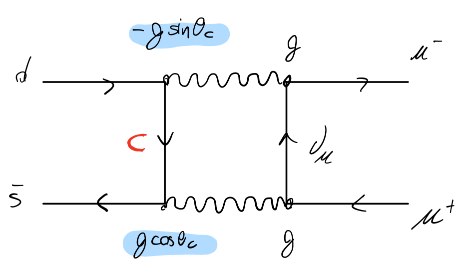


$$\underbrace{\mathcal{M}_u \propto g^4 \cos\theta_c \cdot \sin\theta_c}$$

This is NOT SMALL
$\to$ that's why was expected

So why it does not appear?

Including the CHARM quark ...



Writing down the matrix element

$$\mathcal{M}_c \propto -g^4 \sin\theta_c \cos\theta_c$$

The total amplitude of the decay

$$\text{TOTAL} \quad |\mathcal{M}|^2 = |\mathcal{M}_u + \mathcal{M}_d|^2 = 0$$

because you can not distinguish the states!

But we observe small % of decay $\Rightarrow$ Because we consider quark's mass equal in reality it's not true

THEY PREDICTED THE PRESENCE OF CHARM QUARK

$\rightarrow$ 1974 discovered $|c\bar{c}\rangle$ state called J/ψ

The CKM Matrix

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

The weak charged interaction involving quarks is

$$-i\frac{g}{\sqrt{2}} (\bar{u}, \bar{c}, \bar{t}) \gamma^\mu (1-\gamma^5) \frac{1}{2} \cdot \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

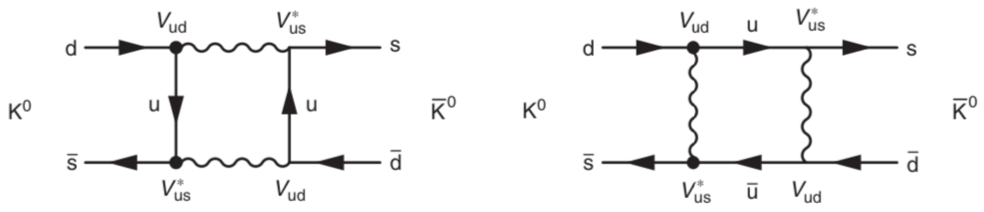
The matrix is unitary $V^\dagger V = \mathbb{1}$

CP violation

The weak interaction not only violates parity but ALSO charge conjugation.

$\rightsquigarrow$ CP violation

Kaon System $\qquad K^0 - \bar{K}^0$ oscillation



Two box diagrams for $K^0 \leftrightarrow \bar{K}^0$ mixing. There are corresponding diagrams involving all nine combinations of virtual up, charm and top quarks.

The quantum state changes in time. We could also have an antiquark below. So 9 possibilities in total

K^0 and $\bar{K}^0$ are QCD eigenstates (we cannot differenciate) because of his oscillat.

There are a lot of stationary states of the K^0-$\bar{K}^0$ system.

Still here we have 2 particular states: K_S^0, K_L^0

$\rightsquigarrow$ They have an big difference in the decay time:

$$\tau(K_S^0) \simeq 0.9 \cdot 10^{-10}\,s \quad \text{and} \quad \tau(K_L^0) \simeq 0.5 \cdot 10^{-7}\,s$$

Theyr masses are close $\quad m(K_S) \simeq m(K_L) \simeq 498$ MeV

Now, IF CP is conserved, QCD eigenstates K_S and K_L corresponds to the CP heigenstates K_1 and K_2

CP eigenstates of the neutral Kaons are

$$K_1 = \frac{1}{\sqrt{2}}(K^0 - \bar{K}^0) \qquad K_2 = \frac{1}{\sqrt{2}}(K^0 + \bar{K}^0)$$

with $CP|K_1\rangle = +|K_1\rangle$ and $CP|K_2\rangle = -|K_2\rangle$

If CP is conserved we should have $|K_2\rangle = |K_L\rangle$ and $|K_1\rangle = |K_S\rangle$ and the only hadronic decay should be:

$$K_S \to \pi\pi$$
$$K_L \to \pi\pi\pi$$

What could be tested to check CP parity?

We could look at the decays (only thing that we can observe)

We can differentiate K_S and K_L with the LIFETIME!

By producing Kaons, and let them oscillate. If we look distant enough, we can differentiate ⇒ Only K_L left at high distance!

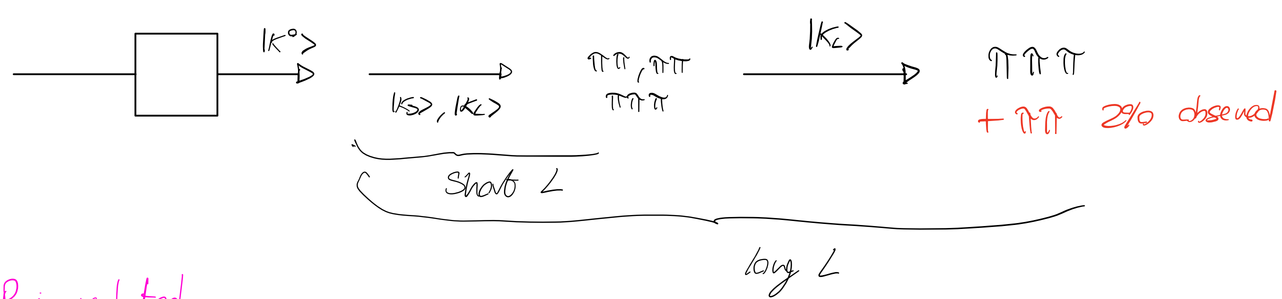
So we should have only 3π in the experiment ⇒ Not happened!

CP violation in K^0 decay

Neutral Kaons are produced with $pp \to K^-\pi^+K^0$ ⇝ K^-, π^+ easily separated using $\vec{B}$.

At the time of the production $|K(t=0)\rangle = |K^0\rangle$ and then K^0 will oscillate in $|K^0\rangle - |\bar{K}^0\rangle$.

This system decay to
$$K_S \to \pi\pi$$
$$K_L \to \pi\pi\pi$$



CP is violated.