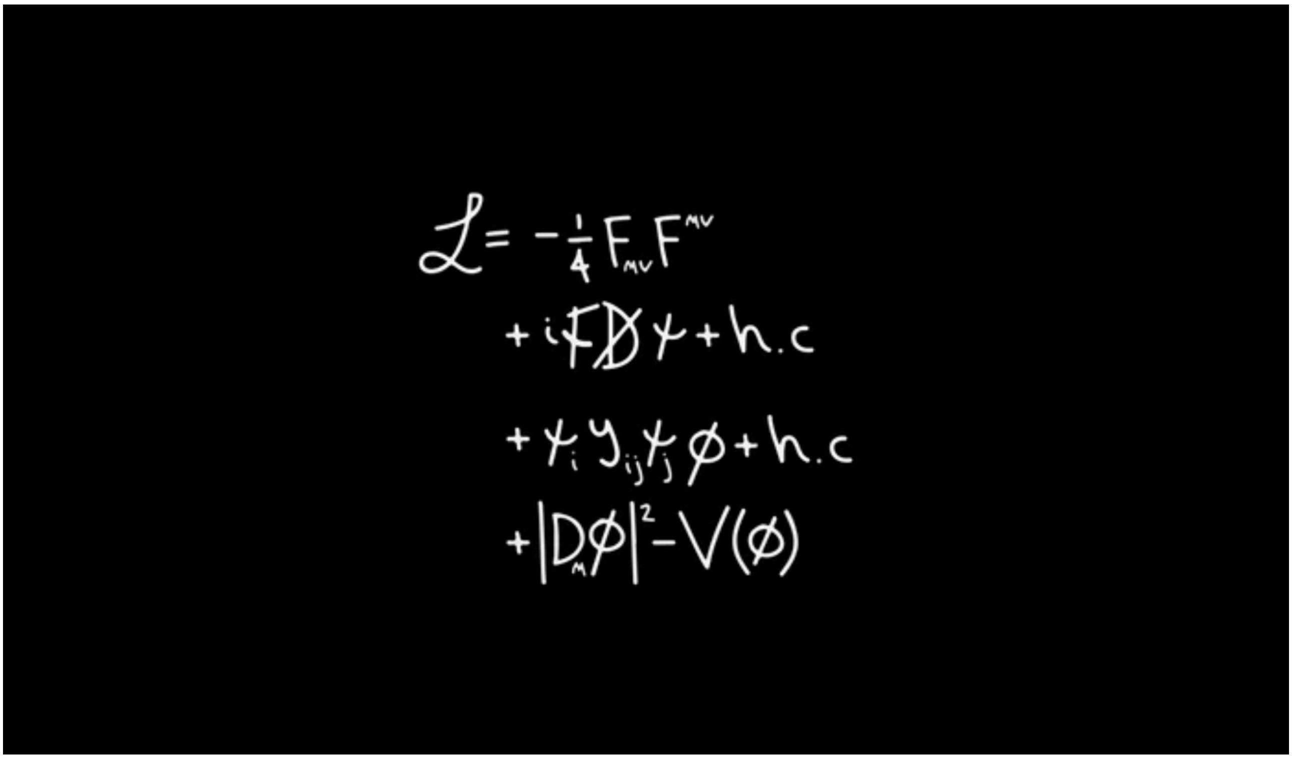
From Standard Model to Beyond the Standard Model



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IMAPP

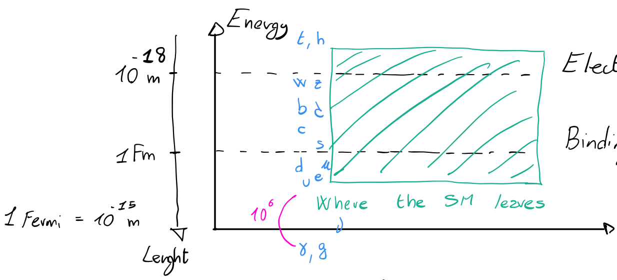
STANDARD MODEL

It's a theory of interactions between matter and forces: strong, weak, electromagnetic. But it doesn't include gravity.

It's a fundamental theory.

What's MATTER? It's quarks, leptons, Higgs.

We tested the theory and we can have a snapshot of the energy.



Electroweak symmetry is broken (Λ_{EWK}) ~ 100 GeV

Binding energy of QCD (Λ_{QCD}) ~ few 100 MeV

(Quantum Mechanics + Special Relat.)

The standard model is a Quantum Field Theory in 3+1 Minkowsky space with (local) gauge symmetry $SU(3)_{col} \times SU(2)_{Left} \times U(1)_Y$

- $SU(3)_{Color}$: Symmetry of the STRONG FORCE of QCD
  Number of generators (Remember $SU(N) = N^2 - 1$) $\Rightarrow$ 8 gluons because $N^2 - 1 = 3^2 - 1 = 8$
  $\rightarrow$ N being the number of generators

- $SU(2)_{Left}$: Symmetry of the WEAK ISOSPIN of EWK
  The force carrier only couples with LH particles
  LH components of particles form a doublet under $SU(2)_L$
  RH componets are SINGLET, they don't interact via Weak Force
  Number of generators 3 : W^+, W^-, Z

- $U(1)_Y$: Describes the EM symmetry group

EW SECTOR

Gauge Symmetry - Example 1: QED

QED, theory of $e^{\pm}, \gamma$

The lagrangian density is:

$$\mathcal{L} = \bar{\psi}\,(i\not{\partial} - m)\psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$\rightsquigarrow F_{\mu\nu} \overset{\text{definition}}{=} \partial_\mu A_\nu - \partial_\nu A_\mu = -F_{\nu\mu}$

- $e^{\pm}$ are spin 1/2 $\quad \psi = \psi(x)$, $\quad \bar{\psi} = \psi^\dagger \gamma^0$ $\quad$ (The field is in the Minkowski space)
- γ are spin 1 $\quad A_\mu = A_\mu(x)$,

The equations of motion:

$e^+, e^- \Longrightarrow \underbrace{(i\not{\partial}} _{\not{p}}- m)\psi = 0 \quad$ Dirac Equation $\quad$ * They have the same units $c = \hbar = 1$

where $\quad i P^\mu = (\overset{\text{energy}}{E}, P_x, P_y, P_z)$

$\gamma \Longrightarrow \partial_\mu F^{\mu\nu} = 0 \quad$ Maxwell Equation

But those particles interact to each other. $\Rightarrow$ The interactions are dictated by SYMMETRY.

Interaction between electron and photon from $U(1)_{QED}$ - Symmetry

So we impose the symmetry by imposing that after the transformation we get a phase factor

$$U(1): \quad \psi(x) \longrightarrow e^{i\alpha(x)}\psi(x)$$

↳ Abelian, unitary group of transformation

The second term of $\mathcal{L}_{free}$ under the transformation is INVARIANT. (we impose the transform)

$$-\bar{\psi} m \psi \longrightarrow \bar{\psi} e^{-i\alpha(x)} m\, e^{i\alpha(x)}\psi = -\bar{\psi} m \psi$$

The first term is NOT!

$$\bar{\psi}(i\not{\partial})\psi \text{ is NOT INVARIANT}$$

$\rightsquigarrow$ We want now to create an invariant term (that cancel out the terms that the ∂ produced)

So we use the COVARIANT DERIVATIVE

$$D_\mu = \partial_\mu + ieA_\mu \quad \text{with} \quad D_\mu\psi \longrightarrow e^{i\alpha(x)}(D_\mu\psi)$$

and so the $D_\mu\psi$ transform as ψ $\Longrightarrow$ Then $\bar{\psi}\, i D_\mu \psi$ is NOW invariant.

Also the photon field transforms, it's a Gauge Transformation

$$U(1): \qquad A_\mu \longrightarrow A_\mu - \frac{1}{4}\partial_\mu \alpha(x)$$

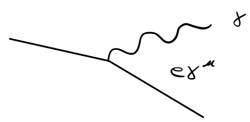
NOTE: it's the SYMMETRY that imposed the interaction! No potential or someting to describe the interaction.

In the end

$$\mathcal{L}_{QED} = \bar{\psi}(i\not{D} - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \quad \text{is } U(1)_{QED} \text{ invariant}$$

Let's take a closer look at the first term

$$\bar{\psi}\, i \not{D}\psi = \underbrace{\bar{\psi}\, i\not{\partial}\psi}_{\text{Kinetic for } e^-} - \underbrace{e\bar{\psi}\not{A}\psi}_{\text{Interaction between } \gamma \text{ and } e}$$



The "predictive" part is that with this theory we can measure things to predict

- m electron mass
- $\alpha_e = \dfrac{e^2}{4\pi} = \dfrac{1}{137}$

} 2 parameters

<u>CONSEQUENCE</u>: Why the photon is massless?

We can write down a potential photon mass term: $\underbrace{m^2 A_\mu A^\mu}_{}$

NOT GAUGE INVARIANT $\Longrightarrow$ $\boxed{m_A = 0}$

This was for QED and for $U(1)$, an ABELIAN GROUP.

Let's try for NOT ABELIAN.

Non Abelian Symmetries

We know from QM that we cannot measure the spin at every direction at the same time with infinite precision.

This is described by SU(2) group : spin.

We know

$$S_{1,2,3} = \frac{\sigma_{1,2,3}}{2} \qquad w/ \quad \sigma_i : \text{Pauli-Matrices}$$

the algebra is $[S_i, S_j] = i\,\varepsilon_{ijk}\, S_k$ for SU(2) || ε_{ijk} : Levi-Civita Tensor

* SU(N) can be described by STRUCTURE CONSTANTS

$$[t_a, t_b] = i \underbrace{f_{abc}}_{\text{Structure constants}} t_c$$

and $N = 2 \quad f_{abc} = \varepsilon_{abc}$

each $t_a \longrightarrow$ one gauge boson

SU(2) - 3
SU(3) - 8
U(1) - 1

$N = 3 \qquad t_a = \frac{\lambda_a}{2} \qquad \lambda_a$: Gell-Mann matrices 3x3 matrices

Gauge Symmetry - Example 2: QCD

Now we require a theory that is invariant under SU(3) local transformations.

$$SU(3): \quad \psi(x) \longrightarrow e^{i\sum_{a=1}^{N^2-1} \alpha^a(x)\, t_a}\, \psi(x)$$

more generators: each generator gives you a different "directions". Each direction it's like an axes in wich you rotate.

The structure of the exponent require $\psi(x)$ to be a vector

$$\psi(x) = \begin{pmatrix} \psi_1(x) \\ \psi_2(x) \\ \psi_3(x) \end{pmatrix} \quad \text{a VECTOR in the color space}$$

So the Lagrangian $\mathcal{L}_{FREE} = \bar{\psi}\,(\underbrace{i\not{\partial}}_{\text{NOT GAUGE INVARIANT}} - m)\,\psi$

As done before, we should now introduce a COVARIANT DERIVATIVE

$$D_\mu = \partial_\mu + i g_s\, G_\mu(x) \qquad w/ \qquad G_\mu(x) = G^a_\mu(x)\cdot t^a$$

→ 8 generators

↳ 8 gluons

strong coupling constants

- $\bar{\psi}(i\not{D})\psi$ is gauge invariant gluon field strenght tensor
- GLUON - GLUON strenght TENSOR

$$G_{\mu\nu} = \partial_\mu G_\nu - \partial_\nu G_\mu \underbrace{+ i g_s [G_\mu, G_\nu]}_{\text{NEW!}} = t^a\, G^a_{\mu\nu}(x)$$

↳ contracting it

NEW! comes from the non abelianity of SU(3)

$$G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu \underbrace{- g_s F^{abc}\, G^b\, G^c}_{\text{NEW PHENOMENOLOGY}}$$

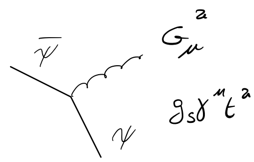
↳ one single

So the lagrangian

$$\mathcal{L}_{QCD} = \bar{\psi}\,(i\not{D} - m)\,\psi - \frac{1}{4}\, G^a_{\mu\nu}\, G_a^{\mu\nu}$$

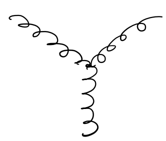
CONSEQUENCES

- gluon is massless due to SU(3) - invariance interactions $\bar{\psi}(i\not{D})\psi$



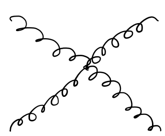
- $G^a_{\mu\nu}\, G_a^{\mu\nu}$ gives rise to self - interactions

G^3 - structure



$g_s \cdot f \cdot G^3 \cdot$ momenta

G^4 - structure



$g_s^2 \cdot f^2 \cdot G^4$

Parameters: quark masses, $\alpha_s = \dfrac{g_s^2}{4\pi}$

Spontaneus Symmetry Breaking

SM: $SU(3)_c \times SU(2)_L \times U(1)_Y \xrightarrow{SSB} QCD \times QED = SU(3)_c \times U(1)_{QED}$

$\rightsquigarrow$ This gives rise to m_W, m_Z, all fermion masses

We know that ψ_L, ψ_R left and right handed fermions $\psi_L \neq \psi_R$.

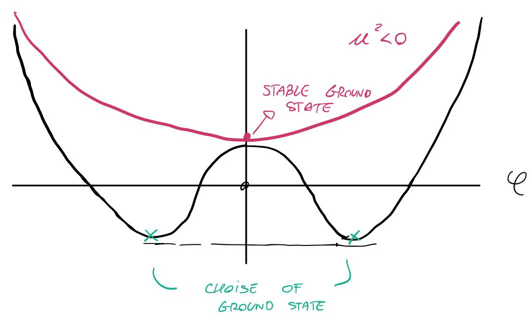
Let's write a Dirac Mass Term $\bar{\psi} m \psi = \bar{\psi}_L m \psi_R + \bar{\psi}_R m \psi_L$ they mix left and right handed field and they brake $SU(2)_L$-gauge invariance.

This violates SYMMETRY $\Longrightarrow$ We need SSB!

Here comes the Higgs Boson φ, spin 0 particle with NO COLOR, but it couples with $SU(2)_L \times U(1)_Y$.

$$\mathcal{L}_{SM} = \bar{\psi} i \not{D} \psi - \Big(\frac{1}{4} \overbrace{F^{\mu\nu} F_{\mu\nu}}^{U(1)} + \overbrace{G^a_{\mu\nu} G^{\mu\nu}_a}^{SU(2)} + \overbrace{\dots}^{SU(3)} \Big) \underbrace{- \bar{\psi} \varphi Y \psi}_{\substack{\text{Yukawa Interactions} \\ \text{(mass term of fermion)}}} \underbrace{- V(\varphi)}_{\substack{\text{Higgs scalar} \\ \text{potential}}} + \underbrace{(D_\mu \varphi)(D^\mu \varphi)^\dagger}_{W,Z \text{ masses}}$$

Let's investigate the potential



- $V(\varphi) = -\mu^2 \varphi^\dagger \varphi + \lambda (\varphi^\dagger \varphi)^2 \quad , \quad \lambda, \mu^2 > 0$

Derivative

$$\frac{\partial V}{\partial \varphi^\dagger} = -\mu^2 \varphi + 2\lambda (\varphi^\dagger \varphi) \varphi = 0$$

$$\varphi = 0 \quad \text{or} \quad \underbrace{-\mu^2 + 2\lambda \varphi^\dagger \varphi = 0}$$

The vacuum expectation value

$$\langle \varphi \rangle \neq 0 = \pm \sqrt{\frac{\mu^2}{2\lambda}}$$

and we know it's 172 GeV

Now we expand the theory around the minimum

$$\varphi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \underbrace{v + h(x)} \end{pmatrix} \longrightarrow \text{Higgs} \quad v = 246 \text{ GeV}$$

φ is complex doublet $= \begin{pmatrix} \varphi^u \\ \varphi^d \end{pmatrix} \rightsquigarrow \in \mathbb{C}^2$ so it has 4 DOF.

But you loose 3 DOF because of SSB (remains only $h(x)$) $\Rightarrow$ $W^{+,-}$, Z eat the 3 DOF and are now able to oscillate in the LONGITUDINAL dof.

FERMION MASSES

Let's see how fermions get masses from Yukawa coupling

$$\bar{\psi} \varphi Y \psi = \bar{\psi} \frac{1}{\sqrt{2}} Y (v + h(x)) \psi$$

$$= \bar{\psi} \underbrace{\frac{Yv}{\sqrt{2}}}_{= m_\psi} \psi + \bar{\psi} \frac{Y}{\sqrt{2}} h(x) \psi$$

So for top quark $m_t \simeq 172$ GeV

$$m_t = \frac{Y_t \cdot v}{\sqrt{2}} \Rightarrow Y_t = \frac{\sqrt{2}\, m_t}{v} \simeq 1$$

Since there are many correlations, v has not been directly measured experimentally.
To know how fermions couple, we have to know how they are charged.

SM matter and their couplings to gauge bosons.

How to memorize the SM Fermions: QUDLE, φ

Q : LH quarks
U : RH quarks, up type
D : RH quarks, down type
L : LH leptons
E : RH leptons

QUARKS	u	c	t	LEPTONS	ν_e	ν_μ	ν_τ
	d	s	b		e	μ	τ
	1°	2°	3°		1°	2°	3°

generations

Notation:

Field name (representation under $SU(3)_L$, representation under $SU(2)_L$)$_Y$ $\rightsquigarrow$ Hypercharge

Example: $Q(3,2)_{1/6}$ $\qquad Q_{ee} = T_3 + Y$

Doublets

$$Q = \begin{pmatrix} u \\ d \end{pmatrix} \qquad \begin{array}{ll} u: & Q_{ee} = +2/3 \,,\ T_3 = +1/2 \qquad Y = Q_{ee} - T_3 - 1/2 = 1/6 \\ d: & Q_{ee} = +2/3 \,,\ T_3 = -1/2 \qquad Y = -1/3 + 1/2 = 1/6 \end{array}$$

Singlets

$U(3,1)_{+2/3}$ $\qquad D(3,1)_{-1/3}$

$L(1,2)_{-1/2}$ $\qquad E(1,1)_{-1}$ $\qquad \varphi(1,2)_{+1/2}$

UNIVERSALITY OF GAUGE INTERACTIONS: identical for all 3 generations $\gg$ gauge interaction are generation blind in SM

Parameters budget:

3 gauge couplings
2 Higgs vectors
3 lepton masses
6 quarks masses $+ \underbrace{3}_{\text{Rotation}} + \underbrace{1}_{\text{Matrix}}$

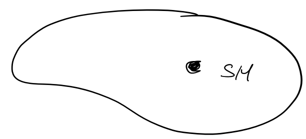
$\Big\}$ 18 parameters

How to move on?

We already mentioned that there are issues with

1) FLAVOR
2) HIGGS, POTENTIAL

$\Big\}$ Portals to NEW PHYSICS



How do we explore other models?

a) Top-down proposals $\Rightarrow$ Strong Theory proposal and check low energy phenomenology.

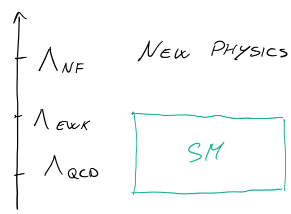
b) Bottom-up proposals $\Rightarrow$ Data driven

c) Effective Field Theory (EFT) $\rightsquigarrow$ Parametrize the unknown

SMEFT : SM effective Field Theory

It's extended version of the SM, so the Lagrangian

$$\mathcal{L}_{SMEFT} = \mathcal{L}_{SM} + \underbrace{\sum_{i,d} \frac{C_i^{(d)} O_i^d}{\Lambda^{(d-4)}}}_{\text{EXTRA TERMS}}$$



$\leftarrow$ Parametrized by extra terms of SM Fields

C_i : Wilson coefficients = dimensionless coupling strenght || O_i : Operators, product of SM Fields and derivatives constants with SM symmetries

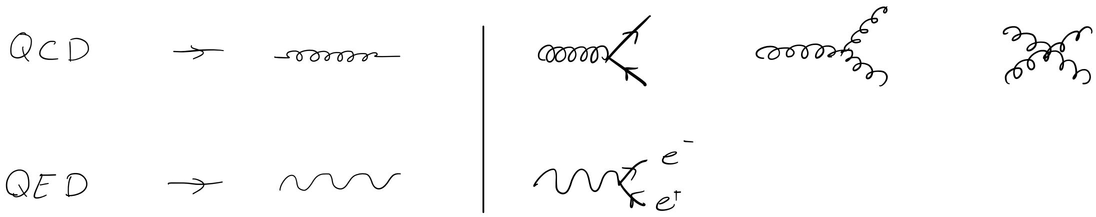
For renormalizeability we cannot have terms like

$$O_{eq} = \bar{Q}\gamma_\mu Q \bar{L}\gamma^\mu L \qquad \text{4 fermions Field}$$

with EFT these terms are allowed and SM is valid only at lower energies.
This is a model independent why to search for new physics.

From Renormalization to Renormalization Group

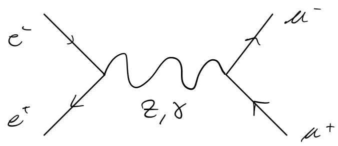
We know about Lagrangians and their Feynman Rules (FR)



We can use those FR to compute scattering (decay matix, cross sections..)

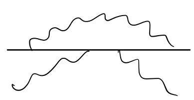
We think of this type of computations as APPROXIMATIONS: PERTURBATION THEORY IN THE couplings

EXAMPLE: $\mu^+\mu^-$ production at LEP → Three level diagram

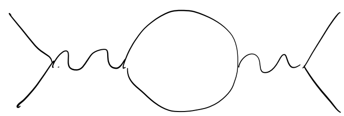


This diagram is an approximation at first order. To increase precision we should go beyond 3-level diagrams ⟹ LOOP DIAGRAMS

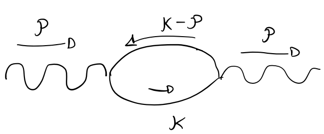
i.e.



i.e



SOMETHIG NEW HAPPENS when we consider the loop diagrams in the calculations. Following the rules we will have



$\rightsquigarrow \underbrace{\int d^4x}_{\text{INTEGRAL}}$

No constrains on k, so mathematically happens that INTEGRAL diverges!

Physically we have a problem: these divergent diagrams seem "naively" to lead to divergences

How to solve?

Idea:

⤳ The couplings that we write in the Lagrangian are not necessarely FINITE, are not physical observables, so we can take the DIVERGENCY and absorb them in a RENORMALIZED couplings

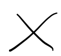
The underlying reason is that the Lagrangian parameters are NOT physical.
In order to make predictions one must express everything in terms of other measurements.

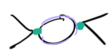
If done, all divergences will cancel.

Example - φ^4

$$\mathcal{L} = \frac{1}{2}\partial_\mu \varphi \, \partial^\mu \varphi - \frac{\lambda}{4!}\varphi^4$$

Compute $\varphi\varphi \to \varphi\varphi$

LO: Leading Order $\longrightarrow i\mathcal{M}^{LO} =$  $= -i\lambda$

NLO: Next to Leading Order $\longrightarrow i\mathcal{M}^{NLO} =$   

s-ch t-ch

Rules: $$i\mathcal{M}_s^{NLO} = (-i\lambda)^2 \, \frac{1}{2}\int \frac{d^4k}{(2\pi)^4} \cdot \frac{1}{k^2} \cdot \frac{1}{(k+p)^2} \qquad p = p_1 + p_2$$

- $i\mathcal{M}$ is a lorenz scalar so it can only depend on a Lorentz scalar, the only one that I have is $s = p^2 = (p_1+p_2)^2 = (k_1+k_2)^2$

- $i\mathcal{M}$ is dimensionless so for large loop momenta $k^\mu \gg p^\mu$

$$i\mathcal{M}^{NLO} \propto \int_0^\infty \frac{d|k|}{|k|} \to \infty$$ which DIVERGES

Now we want to apply the concept said before: REGULATE the divergences parametrizing them.

We expect that (typically if doing calculations)

$$\mathcal{M}_s^{NLO} \propto \log \frac{s}{\Lambda^2}$$ which is the "CORRECT RESULTS" for $\Lambda \to \infty$

energy scale

$[\Lambda] = E$

With Λ we REGULATED the divergence

$\rightsquigarrow$ A trick to compute $\mathcal{M}$ is to compute first its derivative and then integrate

After explicit computation

$$\mathcal{M}(s) = -\lambda - \frac{\lambda^2}{16\pi^2}\frac{1}{2}\log\frac{s}{\Lambda^2} + O(\lambda^3)$$

Renormalization: Express λ in terms of the "Renormalization coupling" that is fixed by experiment.

Fixing λ_R

Experiment @ $s = s_R \to \underbrace{\sigma(s_0)}_{\text{Cross section at } s_0}$: Define $\lambda_R \Rightarrow$ Renormalization condition

$$\lambda_R \equiv \underbrace{-\mathcal{M}(s_0)}$$ extracted from exp, it's a number

- LO $\quad \lambda_R = -\mathcal{M}(s_0) = \lambda$
- NLO $\quad \lambda_R = -\mathcal{M}(s_0) = \lambda + \frac{\lambda^2}{16\pi^2}\frac{1}{2}\log\frac{s_0}{\Lambda^2} + O(\lambda_R^3)$ $\gg$ For $\Lambda \to \infty$, this diverge

NOT s! it's important

To "cure it" $\Rightarrow$ Express λ in terms of Λ

$$\lambda = \lambda_R - \frac{\lambda_R^2}{16\pi^2}\,\frac{1}{2}\,\log\frac{s_0}{\Lambda^2}$$

substituting

$$\mathcal{M}(s) = -\lambda - \frac{\lambda^2}{16\pi^2}\,\frac{1}{2}\,\log\frac{s}{\Lambda^2}$$

$\rightarrow$ This is s now

Plug this in and keep only λ_R^2 terms

$$= -\lambda_R - \underbrace{\frac{\lambda_R^2}{16\pi^2}\,\frac{1}{2}\,\log\frac{s}{s_0}}_{\text{NO DIVERGENCE!}} + \mathcal{O}(\lambda_R^3)$$

$\Rightarrow$ The divergencies of Feynmann diagrams are absorbed in renormalized couplings, which are fixed by experimental observables.

Renormalization Group

Why is renormalization NOT enough?

$\to$ Large logarithms violate the naive scaling and lead to the loss of predictivity.

Concretely: Rate / cross, Γ, with dimension $[\Gamma] = [E^D]$
(observables)

DIMENSION ANALYSIS (means divide Γ by its energy and factorize)

$$\Gamma(E, x, g, m) \overset{D}{=} E^D \, \Gamma\left(1, x, g, \frac{m}{E}\right)$$

$\hookrightarrow$ dimensionless coupling (angles)

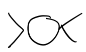
Naive expectation is that

$$\Gamma \xrightarrow{E \gg m} E^D \, \Gamma(1, x, g, 0)$$

This however is NOT what happens!

The log divergencies that we discussed VIOLATE this scaling.

SAME EXAMPLE: φ^4 (with massive φ)

$\mathcal{M} =$  $+$  $+$ crossing

$$= \lambda_0 - \frac{\lambda_0^2}{32\pi^2} \int_0^1 dx \log \frac{\Lambda^2}{m^2 - s(1-x)x}$$

, $\int$ is not easy because mass φ

RENORMALIZE $\lambda_0 \to \lambda_R$

$$= \lambda_R + \frac{\lambda_R^2}{32\pi^2} \int_0^1 dx \left(\log\left[1 - \frac{s\,x(1-x)}{m^2}\right] + \text{crossing} \right)$$

Now take $E \gg m$ (solving)

$$\mathcal{M} \to \lambda_R + \frac{\lambda_R^2}{32\pi^2} \left(\overbrace{\log \frac{-s}{m^2}}^{\text{LARGE LOGARITHM WHEN } s \gg m^2} + \text{crossing} + 6 \right)$$

CONCLUSION:

We cannot understand the $\overset{\text{high energy}}{\text{UV}}$ behaviour of this theory via this fixed order renormaliz. approach

SOLVE: RENORMALIZATION GROUP

The idea is to go back and say that was not a good idea to think g_R as a fixed order expansion

We should think of the dependence as something that depends on the energy but adding a new parameter μ.

$$\Gamma(E, g_R, m_R) \longrightarrow \Gamma(E, g(\mu), m(\mu), \mu)$$

CHANGE PARADIGM

Find a way to define $g(\mu)$ such that for $E \gg m$ it DOES NOT depend on m, but on some new, artificial scale μ.
We can fix $g(\mu)$ with a new condition far away from m, avoiding large logs.

Let's suppose that we did find a way to express $g(\mu)$

NEW SCALING

$$\Gamma(E, g(\mu), m(\mu), \mu) = E^D\, \Gamma\left(1, g(\mu), \frac{m(\mu)}{E}, \frac{\mu}{E}\right)$$

$\rightsquigarrow$ If we care about some energy E, set $\mu = E$

$$\Gamma(E, g(\mu), m(\mu), \mu) \overset{\mu=E}{=} E^D\, \Gamma\left(1, g(E), \frac{m(E)}{E}, 1\right)$$

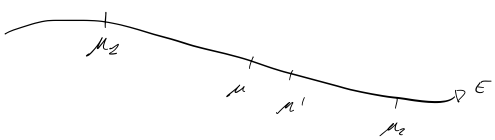
Because per definition $g(\mu)$ doesn't depend on m, this rate does not contain large logs for $E \gg m$ $\Rightarrow$ if we do perturbation theory in terms of $g(E)$

$$\Gamma(E, g(\mu), m(\mu), \mu) \xrightarrow{E \gg m} E^D\, \Gamma(1, g(E), 0, 1)$$

SCALES LIKE THIS

CONCEPT OF RENORMALIZATION GROUP:

The renormalization group absorbs the "anomalous logarithmic" scaling in the renormalized couplings at the price that they depend on a new artificial scale μ.



μ_1, μ_2 are largely separated because of that we cannot compute $g(\mu_2)$ in terms of a fixed order expansion in $g(\mu_1)$. However, we can expand $g(\mu')$ in terms of $g(\mu)$ as long as $\mu' \approx \mu$ (THEY ARE VERY CLOSE)

This allows us to have a corrispondent differential equation such that we can always increment

$\Longrightarrow$ Callan - Symanzik Equation

Differential Equation

$$\boxed{\frac{d g(\mu)}{d \log \mu} = \beta(g)}$$

How does g changes if we change μ

Solution:

$$\log \frac{\mu_2}{\mu_1} = \int_{g(\mu_1)}^{g(\mu_2)} dg \, \frac{1}{\beta(g)}$$

If we know $g(\mu_1)$ at scale μ_1 (where m is not important) we can compute $g(\mu_2)$

The beta Function is actually computeable (From the Feynmann diagrams we computed before)

At $E \gg m^2$

$$g_{BARE} = g(\mu) + \frac{g(\mu^2)}{16\pi^2} \underset{\text{parameter}}{B} \log \frac{\Lambda}{\mu^2} + \dots$$

$\leadsto$ I can do the same with μ'

$\Longrightarrow$ Comparing this with the $g(\mu')$ expansion for $\mu' = \mu \longrightarrow \beta(g)$ in terms of B

$\Longrightarrow$ We can fix the $\beta(g)$ / RGEs by the poles that we absorb in new couplings

RGEs $\to$ Renormalization Group Equations

Asyntotic Scaling Behaviour

Discussing the differential equations 2nd possibility.

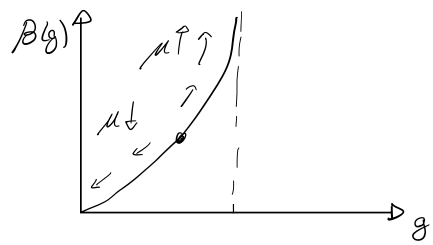
A) Trivial Infrared IR Fixed point (FP) [QED] : means in the InfraRed we get g zero

$\beta(g) > 0$ For small $g \Rightarrow$ in this case I can make an expansion

expansion $\beta(g) = g^2 \beta_0 + \dots$ ⤳ it's expansion in g

So $$g(\mu_2) \overset{\text{1-LOOP}}{=} \frac{g(\mu_1)}{1 - \beta_0 g(\mu_1) \log \frac{\mu_2}{\mu_1}}$$

Obtained by solving the Diff. Equation



- $g(\mu)$ increases with μ increasing and diverges at some Finite value (Landau Pole)

$$\Lambda_{\substack{\text{LANDAU}\\ \text{POLE}}} \overset{\text{1-loop}}{=} \mu_1 \, e^{\frac{1}{\beta_0 \cdot g(\mu_1)}}$$

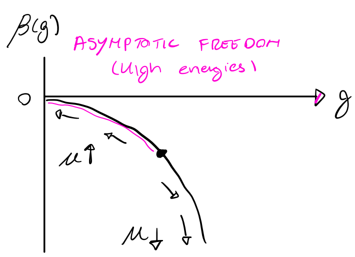
- For $\mu \to 0$, $g(\mu) \to 0 \Rightarrow$ Trivial / Free IR FP

~Trivial FP means that FREE theory with $\beta(g) = 0$ in $g = 0$

B) Trivial UV FP - Asymtotic Freedom [QCD] Nobel Price

For small g $\beta = g^2 \beta_0 +$

↳ Negative with $\beta_0 < 0$



- ASYMTOTIC FREEDOM : μ increasing $\to \beta(g) \to 0$
- When g becomes greater than 1 we lose perturbativity
- Coupling decreases with $\mu \uparrow$ trivial UV FP - asy. Freed
- Coupling grows in the IR

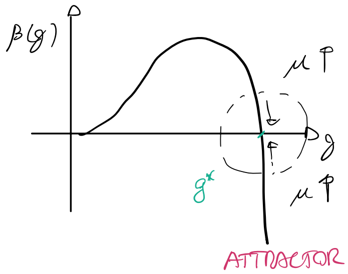
QCD $\alpha_s(\mu_2) = 0.118 \to \begin{cases} \alpha_s(\mu_2) \to 0 & \mu \to \infty \\ \alpha_s(\mu_2) \to \infty & \mu \to 0 \end{cases}$ $\Lambda_{QCD} \simeq 220\,\text{MeV}$

c) NON TRIVIAL FPs

$\exists\, g^* \neq 0$ such that $\beta(g^*) = 0$

$\to$ Place at which for some non-null g the $\beta(g)$ is zero)

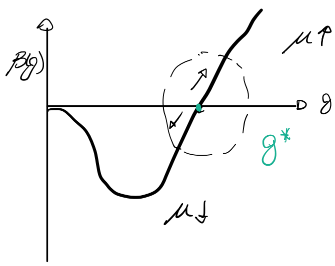
UV STABLE



it's called UV stable cause does not matter of which is μ it goes back to the g^* part

Independent of the starting point in the vicinity of g^* $\quad g(\mu) \xrightarrow{\mu \to \infty} g^*$

STABLE FP



For μ increasing, g is repelled from FP
It is attracted for $\mu \to 0$ (IF STABLE)

Summary

We need the normalization group.

If we try to make predictions far, far away from where we have data.
If we want to know, if we have some data, some objective, some measurement, some energy state, and we now want to make a decision super far away from it, and we try to do this with fixed renormalisation following the regression theory, large logrithms will appear, ruining by the pertubativity and the validity.

We have to rethink: need to define a new coupling that depend on artificial scale does not depend on mass on the higher scale.

Once I have fixed this definition and I've made sure that my renormalized copy, does not depend on this IR scale, okay, I can incrementally compute it at very high scales, very different scales, absorbed all these large numbers in this running,these are the running copy constants, and then I can make a prediction in terms of the fixed following regression theory, but in terms of the high scale copy, no large numbers, no N-square theory.

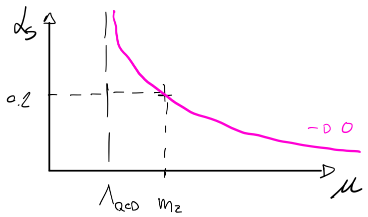
Okay? And because of all this story, this coupling is G, that are running, coupling, they do have some sort of physical interpretation because we can use them as proxies to understand the fate of these theories at the different energy scales, in particular at very high P scales

Running of Wilson Coefficients

In general, most couplings do "run".

$\rightsquigarrow$ their value changes with energy α_s (strong coupling constant)

Ex: α_s



$$\mu \frac{d\alpha_s}{d\mu} = \frac{d\alpha_s}{d\ln\mu} = \beta(\alpha_s, \dots)$$

$\hookrightarrow$ seen previous lecture

All couplings in the SM do run.

Wilson Coeff: Effective Field theory, example: SMEFT

$$\mathcal{L}_{SMEFT} = \mathcal{L}_{SM} + \sum_i C_i^{(d)} \frac{O^{(d)}}{\Lambda^{d-4}}$$

- O_i: higher dimensional operators "vertices" composed out of SM - Fields.
- C_i: wilson coefficients
- Λ: lenght scale

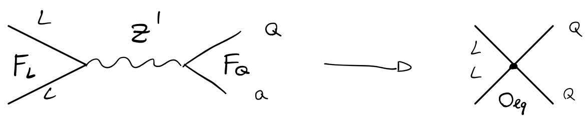
The C_i are dimensionless couplings strenght of operators O_i

Example:

$$O_{lq} = \bar{L}\gamma_\mu L \bar{Q}\gamma^\mu Q$$

(4 - Fermion operator)

UV - model



$m_{Z'} \gg v = 296$ GeV

Computing the Feynmann diagram

$$\bar{L}\gamma_\mu L \frac{\overbrace{g^{\mu\nu}F^Q}^{\text{Propag}}}{q^2 - m_{Z'}^2} \bar{Q}\gamma_\nu Q \simeq -\frac{F_L F_Q}{m_{Z'}^2} O_{lq} = \frac{\overbrace{-F_L F_Q}^{C_i}}{\Lambda^2} O_{lq}$$

at low energies $q^2 \ll m_{Z'}^2 = \Lambda^2$

It's possible to write the equation (of evolution of Wilson coefficients)

$$\boxed{\Gamma_\mu = \frac{dC}{d\mu} = \gamma_C}$$

γ: anomalous dimension (start at loop-level)

Solve this in perturbative theory: Assuming $\gamma^2 = \frac{g^2}{16\pi^2}\gamma^{(0)} + O(g^4)$

Expanding C_i

$$C_i = C_i(\mu, g) = C_i^0(\mu) + \frac{g^2}{16\pi^2} C_i^{(1)}(\mu) + O(g^4)$$

At lowest level QCD ($g = g_s$)

lowest order coefficient

$$\frac{dC^{(0)}}{d\ln\mu} = \underbrace{\frac{g^2}{16\pi^2}}_{= \frac{\alpha_s(\mu)}{4\pi}} C^{(0)}\gamma^{(0)} \implies \boxed{\frac{dC^{(0)}}{C^{(0)}} = \frac{\alpha_s(\mu)}{4\pi}\gamma^{(0)} d\ln\mu}$$

lowest order QCD

From running of α_s at 1 loop: $\frac{d\alpha_s}{d\ln\mu} = \frac{-b_0}{2\pi}\alpha_s^2$ in SM $b_0 = \frac{11 N_c}{3} - \frac{2}{3}N_F > 0$ useful because

$11 - 4 = 7$

$$\iff \alpha_s \, d\ln\mu = \frac{-2\pi}{b_0}\frac{d\alpha_s}{\alpha_s} \implies \frac{dC^{(0)}}{C^{(0)}} = \frac{\gamma^{(0)}}{4\pi}\cdot\frac{-2\pi}{b_0}\cdot\frac{d\alpha_s}{\alpha_s} = -\frac{\gamma^{(0)}}{2b_0}\frac{d\alpha_s}{\alpha_s}$$

Integrate

$$\implies \int_{C^{(0)}(\mu_1)}^{C^{(1)}(\mu_2)} d\ln C^{(0)} = -\frac{\gamma^{(0)}}{2b_0}\int_{\alpha_s(\mu_1)}^{\alpha_s(\mu_2)} d\ln\alpha_s$$

$$\left[d\ln c = \frac{dc}{c} \right]$$

$$\iff \boxed{\frac{C^{(0)}(\mu_2)}{C^{(0)}(\mu_1)} = \left(\frac{\alpha_s(\mu_2)}{\alpha_s(\mu_1)}\right)^{-\frac{\gamma^0}{2b_0}}}$$

RELATION BETWEEN $C^{(0)}(\mu = \mu_1)$ AND $C^{(0)}(\mu = \mu_2)$

Easy example to see that coefficients RUN.

Next computation: In general operators mix, and the anomalous dimension is a matrix.

$\rightarrow$ Here was a number

Operator Mixing

Operators with the same quantum numbers mix. Operators O_i mix onto those with the same or lower mass dimension (d)

$\rightsquigarrow$ General version of what seen before

$$\frac{d\vec{C}}{d\ln\mu} = \gamma^T \vec{C} \qquad \vec{C} = (C_1, C_2, \dots) \quad \text{vector of Wilson coeff}$$

$\hookrightarrow$ TRANSPOSE OF ANOMALOUS DIMENSION MATRIX

$$\gamma = \begin{array}{c|c} & \text{dim5} \;\; \text{dim6} \;\; \text{dim7} \; \dots \\ \begin{array}{c} \text{dim5} \\ \text{dim6} \\ \text{dim7} \\ \vdots \end{array} & \begin{pmatrix} \times & 0 & 0 & 0 & \dots \\ \times & \times & 0 & 0 & \dots \\ \times & \times & \times & 0 & \dots \\ \times & \times & \times & \times & \end{pmatrix} \end{array}$$

MATRIX From dim 5 to ∞

- No entries above the diagonal $\Rightarrow$ OPERATORS MIX ONTO ONES OF EQUAL OR LOWER DIMENSION.

Diagonalize the matrix of anomalous dimensions

$$V^{-1}\gamma^T V = \gamma_D \quad \text{diagonal matrix}$$

V: Matrix that diagonalizes γ^T

$$\gamma_D \stackrel{!}{=} \begin{pmatrix} \gamma_{D1} & & \\ & \gamma_{D2} & \\ & & \gamma_{D3} \end{pmatrix}$$

$$\rightsquigarrow \frac{d\vec{C}}{d\ln\mu} = \underbrace{VV^{-1}}_{\mathbb{1}}\gamma^T \underbrace{VV^{-1}}_{\mathbb{1}} \vec{C} = V \underbrace{V^{-1}\gamma^T V}_{\gamma_D} \underbrace{V^{-1}\vec{C}}_{\vec{C}'}$$

NEW WILSON.

By multiplying V^{-1} on the left $\Rightarrow \dfrac{d\vec{C}'}{d\ln\mu} = \gamma_D \vec{C}'$ Easly solveable

- $\vec{C}' = (C_1', C_2', \dots)$ They evolve independently

Solution at lowest order in QCD

- $$C_j'(\mu_2) = C_j'(\mu_1)\left(\frac{\alpha_s(\mu_2)}{\alpha_s(\mu_1)}\right)^{-\frac{\gamma_{Dj}}{2b_0}}$$

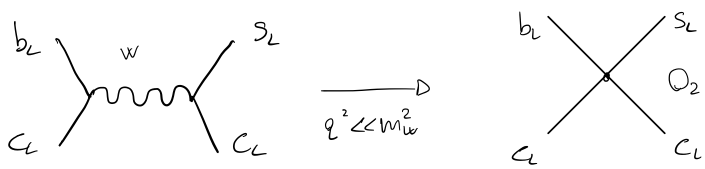
We want to transform this result back in the original basis.

Transforming back: $\quad V^{-1}\vec{C} = \vec{C}' \Leftrightarrow \vec{C} = V\vec{C}'$

explicitely:
$$C_i(\mu_2) = \sum_j V_{ij}\, C_j'(\mu_2) = \sum_j V_{ij}\, C_j'(\mu_1) \left(\frac{\alpha_s(\mu_2)}{\alpha_s(\mu_1)}\right)^{-\frac{\gamma_{0j}}{2b_0}} =$$

$$= \sum_{j,k} V_{ij} (V^{-1})_{jk} \left(\frac{\alpha_s(\mu_2)}{\alpha_s(\mu_1)}\right)^{-\frac{\gamma_{Dj}}{2b_0}} C_k(\mu_1) = C_i(\mu_2)$$

Example: Weak Effective Theory) WET $\quad \mu \leq m_W$



$$O_2 = \bar{s}_{L\alpha} \gamma_\mu c_{L\alpha} \bar{c}_{L\beta} \gamma^\mu b_{L\beta}$$
α, β: color indicies

we can write another operator

$$O_1 = \bar{s}_{L\alpha} \gamma_\mu c_{L\beta} \bar{c}_{L\beta} \gamma^\mu b_{L\alpha}$$
α, β: color indices SWITCHED

At the scale μ_1 $\quad C_1(\mu_1) = 0 \quad C_2(\mu_1) = 1 \quad \to$ induced by W-exchange

$C_1(\mu_1) = 0$ ↓ OFF because W is color blind

Now we want to discover the same operators at μ_2: $C_{1,2}(\mu_2 < \mu_1) = ?$

O_1 and O_2 mix under QCD at 1-loop

$$\gamma^{(0)} = 2 \begin{pmatrix} -1 & 3 \\ 3 & -1 \end{pmatrix} \quad \text{(From books)}$$

$\rightsquigarrow$ Find matrix V that diagonalizes $\gamma^{(0)}$

ANSWER: $\quad V = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = V^{-1}$

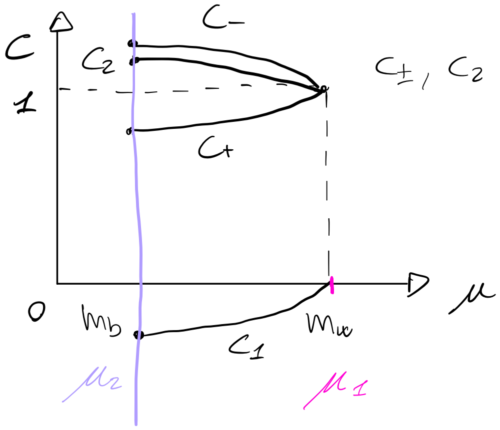
So $\quad \gamma_D = V^{-1} \gamma^{(0)} V = \ldots = \begin{pmatrix} 4 & 0 \\ 0 & -8 \end{pmatrix} \quad$ eigenvalues of $\gamma^{(0)}$

$$\vec{C}' = V^{-1} \vec{C} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} C_1 \\ C_2 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} C_1 + C_2 \\ C_1 - C_2 \end{pmatrix} = \begin{pmatrix} C_1' \\ C_2' \end{pmatrix}$$

C_1' is proportional to $C_+ = C_1 + C_2$ $\longrightarrow$ $C_\pm(\mu_1) = 1$ (since $C_1(\mu_1)=0$, $C_2(\mu_1)=1$)
C_2' is proportional to $C_- = C_2 - C_1$

Eigenvalues : $\gamma_+ = +4$, $\gamma_- = -8$

$$\Longrightarrow C_\pm(\mu_2) = C_\pm(\mu_1)\left(\frac{\alpha_s(\mu_2)}{\alpha_s(\mu_1)}\right)^{-\frac{\gamma_\pm}{2b_0}}$$



- C_+ is reduced because $\frac{\alpha_s(\mu_2)}{\alpha_s(\mu_2)} > 1$ while the experiment is negative
- Now we know C_+ and C_- how evolve but we are interested on C_1 and C_2 at b-physics scale.

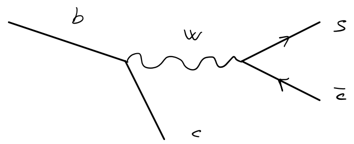
$$C_1(m_b) = -0.3 \qquad C_2(m_b) = 1.1$$

$$C_2 = \frac{C_+ + C_-}{2} \qquad C_1 = \frac{C_+ - C_-}{2}$$

We are sensitive with experiments to this types of effect. SMEFT is the most dominant EFT we can encounter.

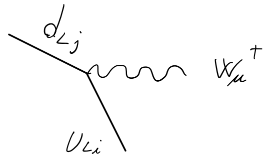
New physics can change W coefficients, that's why one needs to search for new decays.

Effective hamiltonian / Theory for weak decays



Amplitude

Feynman Rule



$\rightsquigarrow \frac{g}{\sqrt{2}} \gamma_\mu L \, (V_{CKM})_{ij}$ $\quad i,j = 1,2,3$

where $L = \frac{1-\gamma_5}{2}$ projector Left-handed Fermions

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$V_{CKM} \in \mathbb{C}$, UNITARY $\quad V_{CKM} \cdot V_{CKM}^\dagger = \mathbb{1}$

$\rightsquigarrow$ V_{CKM} has 4 parameters : 3 angles and 1 CP violation phase

- $\lambda = \sin\theta_c \simeq 0.2$ $\Rightarrow$ Cabibbo angle describe rotation (1 - 2 generation mixing)
- $\lambda^2 \sim$ percent $\longrightarrow$ describe rotation (2 - 3 generation mixing)
- $\lambda^3 \sim$ permille $\longrightarrow$ describe rotation (1 - 3 generation mixing)

$$V_{CKM} \simeq \begin{pmatrix} 1 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 & A\lambda^2 \\ A\lambda^3(1+\rho+i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

with correction $A \simeq \mathcal{O}(1)$

$\rightsquigarrow$ Futher we go away, the smaller becomes

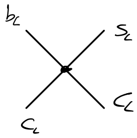
AMPLITUDE

$$= \left(\frac{g^2}{\sqrt{2}}\right)^2 V_{cb} V_{cs}^* \, (\bar{c}_L \gamma_\mu b_L) \underbrace{\frac{g^{\mu\nu}}{q^2 - m_W^2}}_{\text{Propag}} (\bar{s}_L \gamma_\nu c_L)$$

$q^2 \simeq O(m_b^2) \ll m_W^2 \rightsquigarrow$ Neglet q^2 in the propagator

$$\simeq \underbrace{-\frac{g^2}{2m_W^2} V_{cb} V_{cs}^* \underbrace{(\bar{c}_L \gamma_\mu b_L)(\bar{s}_L \gamma^\mu c_L)}_{O_2}}_{A} + \mathcal{O}\left(\frac{g^2}{m_W^2}\right)$$

So $\quad O_2 \simeq (\bar{c}_{L\alpha} \gamma_\mu b_{L\alpha})(\bar{s}_{L\beta} \gamma^\mu c_{L\beta})$ $\quad$ 4 - Fermion operator



Drawing of operator

MATCHING THE SM to the EFFECTIVE THEORY.

$\Rightarrow$ W has been removed from theory $\Rightarrow$ Low energy effective theory "weak effective theory" WET

Read off the Wilson coefficient C_2

$$\mathcal{L}_{eff} = \frac{4 G_F}{\sqrt{2}} V_{cb} V_{cs}^* C_2 O_2 \qquad (B) \qquad \frac{4 G_F}{\sqrt{2}} = \frac{4 g^2}{2 \cdot 8 m_W^2} \quad \rightsquigarrow \text{comparing}$$

$\Rightarrow C_2 = 1$ at $\mu = m_W$ $\quad$ "Matching" by comparing (A) w/ (B)

WE MATCH where we can do this calculation at m_W.

Scale for b-physics phenomenology $\quad \mu = m_b = \mathcal{O}(4\,\text{GeV}) \Rightarrow$ HAVE TO EVOLVE THE Wilson coefficients.

$\Rightarrow$ Take into account the RUNNING + MIXING of operators

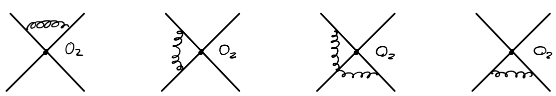
We could write also

$$O_1 = (\bar{c}_{L\alpha} \gamma_\mu b_{L\underline{\beta}})(\bar{s}_{L\beta} \gamma^\mu c_{L\underline{\alpha}}) \qquad \alpha, \beta = \text{color indicies SWAPPED}$$

$C_2(\mu = m_W) = 1$

$C_1(\mu = m_W) = 0$

Contribution to O_1 are induced by QCD - corrections to O_2



$\rightsquigarrow$ Anomalous dimension matrix

$$\gamma = \frac{g_s^2}{16\pi^2} \gamma^{(0)} \quad , \quad \gamma^{(0)} = 2 \begin{pmatrix} -1 & 3 \\ 3 & -1 \end{pmatrix} \qquad \text{(see last lecture)}$$

And including QCD corrections : $\quad \mathcal{L}_{eff} = -\frac{4 G_F}{\sqrt{2}} V_{cb} V_{cs}^* (C_2 O_2 + C_1 O_1)$

leading logarithmic order in α_s

$$C(\mu) = C(m_W) \left(\frac{\alpha_s(\mu)}{\alpha_s(m_W)} \right)^{-\frac{\gamma^{(0)}}{2 b_0}} = C(m_W) \left(\frac{1}{1 + \frac{b_0}{2\pi} \alpha_s(m_W) \log\left(\frac{\mu}{m_W}\right)} \right)^{-\frac{\gamma^{0}}{2 b_0}}$$

Plug formula of α_s RUNNING

expanding parenthesis

can be large, not a good expansion parameter

$$= C(m_W) \left(1 + \frac{\gamma^{(0)}}{2 b_0} \cdot \frac{b_0}{2\pi} \alpha_s(m_W) \log\left(\frac{\mu}{m_W}\right) + O(\alpha_s^2 \cdot \log^2) \right)$$

($\alpha_s \ll 1$ BUT $\alpha_s \cdot \log$ can be large (X) $\to$ α_s is an unstable expansion

(X) : naive, Fixed order expansion uncontrolled, poor control

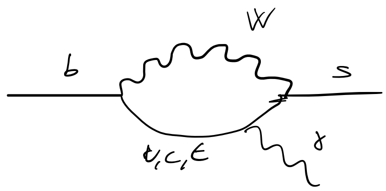
(*) : RG - improved perturbation theory, large logarithms are resummed.

Flavour Changing Neutral Currents

$$\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix}$$

- FCNC's
- charged current

In SM only through loops e.g. $b \to s$



The Flavour changes but not the electric charge

$$A(b \to s\gamma) = \overbrace{\underbrace{V_{tb} V_{ts}^*}_{\lambda^2} f\left(\underbrace{\frac{m_t^2}{m_W^2}}_{\sim 1}\right)}^{u} + \overbrace{\underbrace{V_{cb} V_{cs}^*}_{\lambda^2} f\left(\underbrace{\frac{m_c^2}{m_W^2}}_{\sim 10^{-4}}\right)}^{c} + \overbrace{\underbrace{V_{ub} V_{us}^*}_{\lambda^4} f\left(\underbrace{\frac{m_W^2}{m_W^2}}_{\text{tiny}}\right)}^{t}$$

V_{CKM} is UNITARY $\Longrightarrow$ The GIM mechanism

$$\sum_{i=u,c,t} V_{ib} V_{is}^* = 0 \quad \Longleftrightarrow \quad V_{cb} V_{cs}^* = -\left(V_{tb} V_{ts}^* + V_{ub} V_{us}^*\right)$$

1) If $m_t = m_c = m_u$ then $A(b \to s\gamma) = 0$ (GIM)

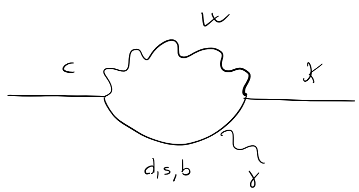
2) If $V_{CKM} = \mathbb{1}$ then $A(b \to s\gamma) = 0$

Writing the terms using the W parametrization

$$A(b \to s\gamma) = V_{tb} V_{ts}^* \left(f\left(\frac{m_t^2}{m_W^2}\right) - f\left(\frac{m_c^2}{m_W^2}\right)\right) + V_{ub} V_{us}^* \underbrace{\left(f\left(\frac{m_u^2}{m_W^2}\right) - f\left(\frac{m_c^2}{m_W^2}\right)\right)}_{\text{sub dominant}}$$

because the difference $m_u - m_c$ compared to m_W is SMALL

For comparison



writing the same thing down

$$A(c \to u\gamma) = CKM\left[f\left(\frac{m_b^2}{m_W^2}\right) - f\left(\frac{m_c^2}{m_W^2}\right)\right] + CKM\left[f\left(\frac{m_d^2}{m_W^2}\right) - f\left(\frac{m_c^2}{m_W^2}\right)\right]$$

Here all the quarks are light so these processes are extremely suppresed and perfect to search for new physics.

What we are actually seeing now is described by 10 parameters : 6 masses of quarks
4 CKM matrix

$A(b \to d\gamma)$ would be equal to $A(b \to s\gamma)$ but substituting $s \to d$ so we would expect order 4 %

Minimal Flavour Violation

All Flavours change structure in the SM (regarding quarks) is described in terms of 10 parameters : 6 quarks masses, 3 mixing angles and 1 phase.

These originate from the Yukawa matrix after electroweak symmetry breaking.

The one from SM is the minimal amount of CP violation we need to explain what we observe.

So expansions of the SM should also leave it. In the SM

$$\mathcal{L}^{SM}_{Yukawa} = -\bar{Q}\, Y_u\, H^c\, U - \bar{Q}\, Y_d\, H D$$

$\rightarrow Y_u, Y_d$ are the SOLO SOURCE of Flavour violation in SM.

A Model is MFV if there are no further sources of Flavour violation

$\Rightarrow Y_u, Y_d$ are also the sole sources of flavour structure too.

We don't know why the masses are like that. And other stuff.

MFV: Minimal Flavour Violation, benchmark for SM-extensions

"No more flavour-violation arises only in Yukawa

$$\mathcal{L}^{SM}_{Yukawa} = -\bar{Q} H^c Y_u U - \bar{Q} H Y_d D + h.c.$$

In the hypothetical limit $Y_u = Y_d = 0$ the SM has ONE more symmetry (FLAVOUR SYMMET)

Symmetry G_q: $SU(3)_{Q_L} \times SU(3)_{U_R} \times SU(3)_{d_R}$

Q_L: weak DOUBLETS ~> LH quarks
U_R: weak singlets ~> RH up-type quarks
d_R: weak singlets ~> RH down-type quarks

Recap:

$$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \begin{pmatrix} c_L \\ s_L \end{pmatrix}, \begin{pmatrix} t_L \\ b_L \end{pmatrix} \qquad U_R = u_R, c_R, t_R \qquad D_R = d_R, s_R, b_R$$

each set of quarks transform as a triplet under CORRESPONDING SU(3)
Example: Q_L, U_R, D_R

$$\begin{aligned} Q_L &\to V_Q Q_L \\ U_R &\to V_U U_R \\ D_R &\to V_D D_R \end{aligned} \qquad \text{where} \quad V_Q \in SU(3)$$

unitary 3x3 matrices
They live in generation space

The thing is that with $SU(3)_{Q_L}$, the triplet is composed of single doublets q^1, q^2, q^3
THE MATRICES ROTATE THE GENERATIONS => NO DIFFERENCE

=> Quark Fields transformations under G_q

$$\underset{SU(3)_Q}{(\;\cdot} \quad \underset{SU(3)_U}{\cdot} \quad \underset{SU(3)_D}{\cdot\;)}$$

$Q(3,1,1)$ $\qquad U(1,3,1)$ $\qquad D(1,1,3)$

Gauge Fields + Higgs are singlets under G_q

⇒ Now we want to switch ON Yukawa, so we write $\mathcal{L}_Y^{SM}$ in an invariant way under G_q

How:

Promote Yukawa couplings to "SPURION" - fields

$$Y_U\,(3,\bar{3},1) \qquad Y_D\,(3,1,\bar{3})$$

Formally invariant terms can be written as

$$\mathcal{L}_{YuK} = \bar{Q}_L H Y_D D_R + \underline{\bar{Q} H Y_U U}$$

- $\bar{Q} Y_U U$ invariant under G_q? Check if it's invariant under single SU(3) groups

$SU(3)_D$? Trivial because singlets here ✓
$SU(3)_U$? $\bar{3} \times 3 \simeq 1$ it's singlet when combined ✓
$SU(3)_Q$? $\bar{3} \times 3 \simeq 1$ it's singlet again

} $\mathcal{L}$ is invariant under G_q

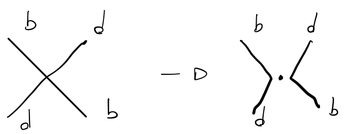
~ Under the assumption

- $\bar{Q} Y_d D$ analogously

$\mathcal{L}^{NP}$ is MFV if there are no further sources of flavour symmetry breaking then the Y_u, Y_d

⟶ Strong constraint, many viable models, very predictive

Example 1: B - $\bar{B}$ mixing



FCNC current:

$$\bar{Q}_i X_{ij} Q_j \rightsquigarrow \begin{matrix} \bar{d}_L X_{13} b_L \\ \bar{s}_L X_{23} b_L \\ \bar{b}_L X_{23} b_L \end{matrix}$$

MATRIX w/ FLAVOUR STRUCTURE

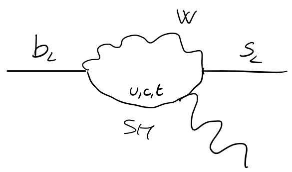
in SM happens at loop level ⇒ Suppressed

In this theory Yukawa controls the flavour mixing.

X_{ij} structure is determined in MFV: $\bar{Q} X^{MFV} Q$ must be invariant under G_q and be accomplish using this ansatz

$$X^{MFV} = \underbrace{a\,\mathbb{1}}_{\text{NO FCNS's}} + \underbrace{b\, Y_u Y_u^\dagger}_{\text{NO FCNS's}} + \underbrace{c\, Y_d Y_d^\dagger}_{\text{NO FCNS's}} + \text{higher order of Yukawa couplings}$$

Example 2: $b \to s\gamma$



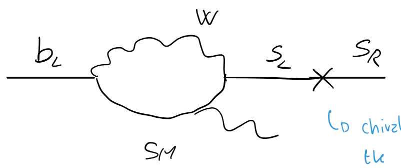
operator $\quad \bar{Q}_i H \,\sigma_{\mu\nu} D_j F^{\mu\nu}$ $\quad$ w/ $\sigma_{\mu\nu} = \frac{1}{2}[\gamma_\mu, \gamma_\nu]$

↳ dim 6 $\qquad F^{\mu\nu}$ = EM-Field strenght tensor

To get the contribution we need to mix Righ Handend and Left Handend quarks.

$\Rightarrow$ In the operator appears D_j (For RH quarks)

We pay the price of a mass term and we do chirality flip of s



$\sim m_s$

↳ chirality flip on the strange quark

Structure in flavour space?

Fermion structure for dipole operator

$$\bar{Q}_i \, Z_{ij}^{MFV} D_j$$

↳ connects LH and RH quarks

Go to mass eigenstate basis Y_D it's diagonal

- $Y_D = \mathrm{diag}\left(\frac{m_d}{v}, \frac{m_s}{v}, \frac{m_b}{v}\right)$ (We are studying FCNC among down types quarks)

$$= \begin{pmatrix} \ddots & & 0 \\ & \ddots & \\ 0 & & \ddots \end{pmatrix}$$

- $Y_U = V_{CKM}^\dagger \tilde{Y}_U \quad$ w/ $\tilde{Y}_U = \mathrm{diag}\left(\frac{m_u}{v}, \frac{m_c}{v}, \frac{m_t}{v}\right)$

$\to$ The Higgs will get its VEV v

$\to$ we know that at three level NO FCNC

$$Z_{ij}^{MFV} = \alpha \underbrace{Y_d}_{\text{NO FCNC}} + \beta \underbrace{Y_U Y_U^\dagger Y_d}_{\text{First NON TRIVIAL term}} + \dots$$

leading contribution in MFV-models to $b \to s\gamma$

The operator will be

$$v\,(d_L)_i\,(Y_u Y_u^\dagger \underline{\underline{Y_D}})_{iK}\,\sigma_{\mu\nu}\,(d_R)_K\,F^{\mu\nu}$$

case A: $i=2,\ k=3$ "$b_R \to s_L\gamma$" $\Rightarrow v\cdot\underline{\underline{\frac{m_b}{v}}}\ \bar{s}_L\,(Y_u Y_u^\dagger)_{23}\,\sigma_{\mu\nu}\,b_R\,F^{\mu\nu}$

case B: $i=3,\ k=2$ "$b_L \to s_R\gamma$" $\Rightarrow v\cdot\underline{\underline{\frac{m_s}{v}}}\cdot b_L\,(Y_u Y_u^\dagger)_{32}\,\sigma_{\mu\nu}\,s_R\,F^{\mu\nu}$

$\frac{B}{A}$ is $\frac{m_s}{m_b}$ $\Downarrow$ suppressed in SM

Generic in MFV

What does $(Y_u Y_u^\dagger)_{23}$ mean?

We can test experimentally this hierarchy

$$Y_u Y_u^\dagger = V^\dagger \tilde{Y}_u \tilde{Y}_u^\dagger V \qquad \text{using } V_{CKM} = V$$

In components

$$(Y_u Y_u^\dagger)_{iK} = (V^\dagger)_{il}\,(\tilde{Y}_u^2)_{ll}\,(V)_{lK}$$

$\to$ already diagonal

$$\simeq y_t^2\, V_{3i}^*\, V_{3K} + \text{terms with } \underline{\text{lighter masses}}$$

$\to$ dominated by top

$$\sim\!\!> \quad Y Y_u^\dagger = y_t^2 \begin{pmatrix} \lambda^6 & \lambda^5 & V_{td}^* V_{tb} \\ h.c. & \lambda^4 & V_{ts}^* V_{tb} \\ h.c. & h.c. & 1 \end{pmatrix} \qquad \left[V \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ -\lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \right]$$

We can now COMPARE $\frac{b \to s\gamma}{b \to d\gamma}$ as $\frac{V_{ts}^* V_{tb}}{V_{td}^* V_{tb}} \sim \frac{1}{\lambda} = \frac{V_{ts}^*}{V_{td}^*}$

This statement could be enlightment for us
we can have us but the flavour sector behaves like in the SM: it's a parametrization of flavour doesn't explain flavour.

Froggat - Nielson - Symmetries

Model of Flavour : extra $U_{FN}(1)$-symmetry

We start again from Yukawas $\bar{Q}_i H^c Y_{u_{ij}} U_j + \bar{Q}_i H Y_{d_{ij}} D_j$

assign Q, U, D generation dependent charges under $U_{FN}(1)$

(Higgs unchanged under $U(1)_{FN}$)

This is what we want to break in order to create flavour.
We need an extra ingredient to get this invariant under $U(1)$:

$\Longrightarrow$ Add scalar field "Flavour" S with charge -1 under $U(1)_{FN}$, UNCHARGED under SM gauge interact.

Field Ψ_i	CHARGES $q(\Psi_i)$
S	-1
H	0
$\bar{Q}_1$	4
$\bar{Q}_2$	2
$\bar{Q}_3$	0
U_1	4
U_2	2
U_3	0

plus D_1 D_2 D_3

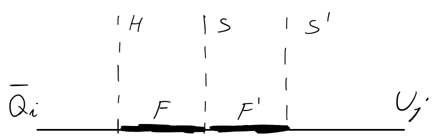
the following terms are $U(1)_{FN}$ - invariant

$$\bar{Q}_3 H^c U_3 + \bar{Q}_3 H^c S^2 U_2 + \ldots \bar{Q}_i H^c S^{q(\bar{Q}_i) + q(U_j)} U_j$$

Now we break $U(1)_{FN}$ spontaneously $\to$ How? S Field gets a non trivial V.E.V.
$\langle S \rangle \neq 0$

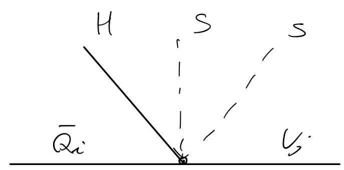
$\to$ Let's think about how could this work as a real model

The fully UV-theory needs also more heavy fermions



F and F' are heavy,
S has a mass M
mass $M \gg \langle H \rangle$

At low energy



When S gets a VEV we can replace (at low energy) the S field with $\frac{\langle s\rangle^2}{M^2}$ (only relevant for phenomenology)

This model can be fitted to data and we can find the values for the Yukawa couplings. We replace S in the lagrangian with $\frac{\langle S\rangle}{M} = \varepsilon$

~ where ε is of the order of Wolfenstein's parameters.

$$Y_{u,d} \sim \left(\frac{\langle s\rangle}{M}\right)^{q(\bar{Q}_i) + q\binom{u_j}{d_j}}$$

So U_{FN} model predicts the hierarchy in the Yukawa up to factors of order 1

So the required ingredients at low energies are $q(\psi_i)$, ε with $\varepsilon \simeq 0.2$

$U(1)_{FN}$ (This is a postulational + generation-dependent charges q_i to the quarks + 1 scalar field S that spontaneously breaks the symmetry
mass M

If we write the Yukawa-lagrangian as an effective theory

$$(Y_{u,d})_{ij} \sim \left(\frac{\langle s \rangle}{M}\right)^{q(\bar{Q}_i) + q\binom{U_j}{D_j}}$$

For low energy phenomena, we only focus on $\varepsilon = \frac{\langle s \rangle}{M}$; $\varepsilon \sim 0.2$ (of the order of the Wolfenstein parameter $\lambda \simeq \sin\theta_c$

We can only predict HIERARCHIES, in terms of ε, not values (because we are not building the full model). So FN-symmetry predicts the hierarchy in terms of ε.

This is very succesful.

From last lecture, we change

$q(\bar{Q}) = (4, 2, 0) = q(U)$, $X_{ij} = q(\bar{Q}_i) + q(U_j)$ our experiment, we can construct now Y_U matrix

$$Y_U \sim \begin{pmatrix} \varepsilon^8 & \varepsilon^6 & \varepsilon^4 \\ \varepsilon^6 & \varepsilon^4 & \varepsilon^2 \\ \varepsilon^4 & \varepsilon^2 & \varepsilon^0 = 1 \end{pmatrix} = \{\varepsilon^x\}$$

→ prediction from $U(1)_{FN}$ symmetry

EXAMPLE Lower dimension: 2 families and 2+3 generations

$$Y_U \sim \begin{pmatrix} \varepsilon^4 & \varepsilon^2 \\ \varepsilon^2 & 1 \end{pmatrix} \qquad Y_d \sim \begin{pmatrix} \varepsilon^3 & \varepsilon^3 \\ \varepsilon & \varepsilon \end{pmatrix}$$

We have to diagonalize these matrices by using perturbation theory, as customary in QM. This perturbative diagonalization gives eigenvectors and eigenvalues in powers of ε.

The eigenvalues are

$$\frac{m_c}{m_t} \simeq \varepsilon^4 \qquad \frac{m_s}{m_b} \simeq \varepsilon^2 \qquad \underbrace{\theta_U \simeq \varepsilon^2 \quad \theta_d \simeq \varepsilon^2}_{|V_{cb}| \sim \varepsilon^2}$$

There can be flavor structure in general, but with this symmetry we can use the following example to calculate the BSM flavour structure.

A) SUPER SYMMETRY: Minimal Supersymmetry Standard Model (MSSM)

QUDLE (SM) $\longrightarrow$ scalar partners $\tilde{Q}, \tilde{U}, \tilde{D}, \tilde{L}, \tilde{E}$

we have 3 generations of quarks $\tilde{Q} = (\tilde{Q}_1, \tilde{Q}_2, \tilde{Q}_3)$

But we are interested in mass term for our new scalars.

$$\mathcal{L}_{MASS} \supset \tilde{Q}_i^+ (m_{\tilde{Q}}^2)_{ij} \tilde{Q}_j + \ldots \underline{\text{mass term}} \text{ of scalars}$$

$\hookrightarrow$ MASS MATRIX (hermitian). 3x3 and complex

We want to use the Froggatt-Nielsen technique to infer the structure of this BSM matrix

$\longrightarrow$ Predict $m_{\tilde{Q}}^2$ using $U(1)_{FN}$ - symmetry

$$m_{\tilde{Q}}^2 \sim \bar{m}^2 \begin{pmatrix} 1 & \varepsilon^2 & \varepsilon^4 \\ h.c & 1 & \varepsilon^2 \\ h.c & h.c & 1 \end{pmatrix} \sim \bar{m}^2 \, \varepsilon^{q(\bar{Q}_i) + q(Q_j) - q(\bar{Q}_j)}$$

we want to construct terms that are allowed so everything is set by ε.

B) SMEFT

Operations with Fermions carry indices

$$\bar{Q}_i \gamma_\mu Q_j \varphi^T \overleftrightarrow{D}^\mu \varphi$$

$\longrightarrow$ containing the quarks $(\bar{Q}_i, Q_j)$, the higgs φ and a covariant derivative

its Wilson coefficients C_{ij} : 3x3 hermitian matrix of W. Coefficients
(Assuming $U(1)_{FN}$ it's possible to structure out the W. coefficients)

Assuming $U(1)_{FN}$

$$C_{ij} \sim \bar{C} \begin{pmatrix} 1 & \varepsilon^2 & \varepsilon^4 \\ h.c & 1 & \varepsilon^2 \\ h.c & h.c & 1 \end{pmatrix}$$

it's the same structure as in MSSM because the $\mathcal{L}$ term is linear in Q and $\bar{Q}$

(How to get structure with no hierarchy things $\to$ neutrinos)

CKM, quark masses: hierarchical, explain using FN- symmetries.

Neutrino sector: PMNS 3×3 matrix, unitary, 3 angles, 1 CP violating phase [+ 2 Maj. Phases]
(if they are Dirac particles the same "machine" of quark sector we have)

PDG: $\theta_{12} \simeq 0.59$, $\theta_{23} \simeq 0.84$ } (larger than 1-2 gen mixing angle of quark: θ_c)

$\theta_{13} \simeq 0.2$ (order of θ_c)

3 neutrino mass eigenstates m_1, m_2, m_3

NORMAL HIERARCHY (NH): $m_1 < m_2 < m_3$
(when we measure are the oscillations so connected to the mass differences)

$$\Delta m^2_{32} = \pm 2.5 \cdot 10^{-3}\,eV \qquad \Delta m^2_{12} = 7.5 \cdot 10^{-5}\,eV^2$$

Inverted Hierarchy (IH): $m_3 < m_1 < m_2$

This is all what we know of neutrinos

How to get the PMNS from symmetries when is not hyerarchical?

Before to start, situation about PMNS: A) $\theta_{13} \ll \theta_{12} \sim \mathcal{O}(1)$
B) $\theta_{13}, \theta_{12}, \theta_{23} \sim \mathcal{O}(1)$

both means are viable but involve different flavour model building

A) $\to$ today, discrete symm
B) "anomaly"

Wee'll follow the case A

PMNS : U "tribimaximal" (MAXIMAL MIXING AMONG ALL THREE FLAVOUR STATE)

$$U = \begin{pmatrix} \sqrt{2/3} & \sqrt{1/3} & 0 \\ \sqrt{1/6} & \sqrt{1/3} & -\sqrt{1/2} \\ -\sqrt{1/6} & \sqrt{1/3} & \sqrt{1/2} \end{pmatrix}$$

storical (very predictive)
ANSATZ predicting 0 have
Not the full picture:
need correction to obtain a value θ_{13}

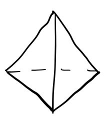
$\rightarrow$ Non hierarchical structures follow from non-abelian, discrete finite number of elements

Examples:

S_n : group of permutations of n objects

$n=3$ $e\mu\tau$ $\tau e\mu$ $\mu e\tau$ $e\tau\mu$ $\tau\mu e$ $\mu\tau e$ } 6 elements (can be wrapped into each other)

A_4 : tetraheder, 12 elements



4 corners
4 surface
$\rightarrow$ has a high symmetry under rotation by 120° and reflation (180°)

REPRESENTATION : singlets (1 dimensional) : ℓ, ℓ', ℓ''
triplets (3 dimensional) : 3 symmetric
3' (anti symmetric)

multiplication rules (to construct mass)

$\ell - \ell = \ell$
$\ell - \ell' = \ell'$
$\ell - \ell'' = \ell''$
$\ell' - \ell'' = \ell$

Triplets: $a = (a_1, a_2, a_3)$ $b = (b_1, b_2, b_3)$
$\ell = (a,b) = a_1 b_1 + a_2 b_3 + a_3 b_2$ (transforms as a singlet)

$\ell' = (ab)' = a_3 b_3 + a_1 b_2 + a_2 b_1$
$\ell'' = (ab)'' = a_2 b_2 + a_1 b_3 + a_3 b_1$

understanding these structures, how they do transform, we can construct invariances?

	L	e_R	μ_R	τ_R	φ_S	φ_T	ξ
A_4	3	1	1''	1''	3	3	1

SPURIONS, responsable for the breaking of the Flavour symm

$$\mathcal{L}_e = y_e \bar{e}_R \frac{H}{\Lambda} (\varphi_T \cdot L) + y_\mu \bar{\mu}_R \cdot \frac{H}{\Lambda}^T (\varphi_T \cdot L)' + y_\tau \bar{\tau}_R \frac{H}{\Lambda} \cdot (\varphi_T \cdot L) + \text{h.c}$$

where Λ is the scale of A_4 breaking: $\langle \varphi_T \rangle = (V_T, 0, 0)$

$\Rightarrow \mathcal{L}_e$ should transform as a $\mathcal{L}$ under A_4

In order for the first term to transform as $\mathcal{L}$ the product of triplets $\varphi_T \cdot L$ must transform as a one.

We can do the same reasoning for all the other terms using the multiplication rules.
We obtain

$$\langle \varphi_T \cdot L \rangle = \langle \varphi_{T1} L_1 + \varphi_{T3} L_2 + \varphi_{T2} L_3 \rangle = V_T L_e = V_T \cdot L_e$$

$$\langle \varphi_T \cdot L \rangle' = \qquad = V_T \cdot L_\mu$$

$$\langle \varphi_T \cdot L \rangle'' = \qquad = V_\tau \cdot L_\tau$$

Finally

$$Y_e = \frac{V_\tau}{\Lambda} \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix} \rightarrow \text{DIAGONAL !}$$

So in this model, PMNS - rotation from neutrino sectors

Recall the neutrino masses come from the see-saw mechanism

$$h_\nu = x_a \xi \frac{LLHH}{\Lambda^2} + x_b (\varphi_S LL) \frac{H^\dagger H}{\Lambda^2} + \text{h.c.}$$

If we work this out we would get the matrix U, the tribimaximal PMNS matr.

The procedure would be to compute $LL = 3$ 2nd then $\langle \varphi_s \rangle LL$ where $\langle \varphi_s \rangle = (v_s, v_s, v_s)$, $\langle \xi \rangle = u$

As intermediate step is

$$M_\nu = M_\nu^T = \frac{\langle u \rangle^2}{\Lambda} \begin{pmatrix} a + \frac{2}{3} b & -\frac{1}{3} b & -\frac{1}{3} b \\ -\frac{1}{3} b & -\frac{2}{3} b & -\frac{1}{3} b \\ -\frac{1}{3} b & -\frac{1}{3} b & \frac{2}{3} b \end{pmatrix} \qquad a = 2x_a \frac{u}{\Lambda} \qquad b = 2x_b \frac{v_s}{\Lambda}$$

Diagonalizing

$$U^\dagger M_\nu U = \frac{\langle M \rangle}{\Lambda} \operatorname{diag}(a+b, a, b-a)$$

where U = Tribimaximal. It's close to reality although it needs corrections.