Maltoni

7) Consider the optical theorem and explain using examples From phenomenology oF Fundamental interactions (SM, BSM, EFT) when one expects unitarity to be violated or not at some scale.

We are gonna prove the optical theorem

$$\int |\psi(x,t)|^2 dx = 1$$ in quantum mechanics

↳ probability density (PDF)

This integral is 1 at time 0 and at every time t

- $\langle \psi, t | \psi, t \rangle = \langle \psi, 0 | \psi, 0 \rangle$ UNITARITY: conservation of probability
- $|\psi, t\rangle = e^{-iHt} |\psi, 0\rangle$ $H = H^\dagger$

When H is hermitian there is no decay in the system (energy is conserved)

$$|\psi, t\rangle = e^{-iHt} |\psi, 0\rangle$$

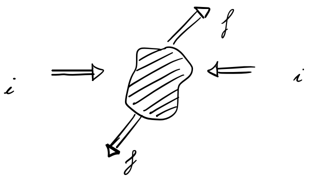
evolution operator

$$S = e^{-iHt} \to S^\dagger S = 1$$ EVOLUTION IS UNITARY

Hence in QFT is the same. Defining the scattering matrix S

$$\langle f | S | i \rangle = (2\pi)^4 \delta^4(p_f - p_i) \mathcal{M}(i \to f)$$

i and f are initial and Final states ~> states where there is no interaction



$\mathcal{L}$

↳ Not a physical object
we can always make a transformation

$\mathcal{M}$ → is the physical object, it doesn't change and is what we measure

$\mathcal{L}$ is a generator For $\mathcal{M}$ but not a physical object ~> is more like how we describe the physical obj

Let's try now with a non hermitian operator

$$S = \mathbb{1} + iT$$

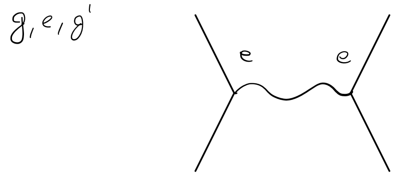
$\hookrightarrow$ NON TRIVIAL and NON HERMITIAN

Hence

$$(\mathbb{1} - iT^{+})(\mathbb{1} + iT) \overset{\text{since } S^{+}S = \mathbb{1}}{=} \mathbb{1} \longrightarrow i(T - T^{+}) = T^{+}T$$

$\hookrightarrow$ is telling me something about the Im part of the scattering amplitude and the square of it!

We never know S exactly but we Taylor expand it T in the coupling and each expansion has a power in the coupling



with the previous $\underline{\underline{\text{equation}}}$ we are connecting loops with the tree level computations

$\Rightarrow$ It's important and not trivial cause connects tree level with higher order

Let's now use the relation in a computation of a scattering amplitude

$$LH: \langle f | i(T - T^{+}) | i \rangle = i \langle i | T | f \rangle^{*} - i \langle f | T | i \rangle$$

$$\overset{!}{=} i (2\pi)^4 \delta^4(\Sigma p) \left[\mathcal{M}^{*}_{f \to i} - \mathcal{M}_{i \to f} \right]$$

$$RH: \langle f | T^{+} T | i \rangle$$

Remember: $\langle \psi_1 | \psi_2 \rangle = \langle \psi_1 | \mathbb{1} | \psi_2 \rangle$ and $\mathbb{1} = \sum_x \int d\Pi_x |x\rangle\langle x|$

"Resolution of identity"

So

$$\langle f | T^{+} T | i \rangle = \sum_x \int d\Pi_x \underline{\langle f | T^{+} | x \rangle} \underline{\langle x | T | i \rangle} =$$

$$= \sum_x \underline{(2\pi)^4 \delta^4(p_f - p_x)} \, \underline{(2\pi)^4 \delta^4(p_i - p_x)} \cdot \int d\Pi_x \underline{\mathcal{M}(i \to x)} \, \underline{\mathcal{M}^{*}(f \to x)}$$

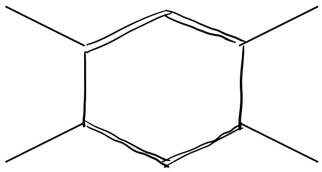
we now equal the 2 members

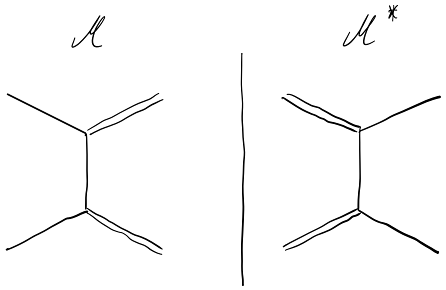
$\delta^4(p_f - p_x)\,\delta^4(p_i - p_x)$ is like 2 deltas for $p_F = p_i$ and one of them is gone cause on both sides while the other is left

$$\boxed{\mathcal{M}(i\to f) - \mathcal{M}^*(f\to i) = i\sum_x \int d\Pi_x\,(2\pi)^4\delta^4(p_f - p_i)\,\mathcal{M}_{i\to x}\,\mathcal{M}^*_{f\to x}} \qquad (*)$$

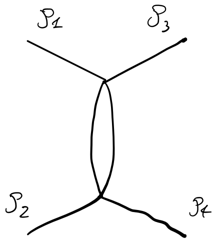
⇒ This is what is implied by UNITARITY: we are linking something linear in $\mathcal{M}$ with something quadratic in $\mathcal{M}$

We are RELATING this 2 :





Let's consider the 2 → 2 scattering



$$s = (p_1+p_2)^2 = (p_3+p_4)^2$$
$$t = (p_1-p_3)^2 = (p_2-p_4)^2$$
$$u = (p_1-p_4)^2 = (p_2-p_3)^2$$

3 invariants

$$s+t+u = \sum m_i^2 = 0$$

related by 4-momentum
⇒ 2 INDEPENDENT VARIABLES

$\underline{\underline{s}}$ $\quad t = -\frac{s}{2}(1-\cos\theta)$ $\qquad \underline{\underline{\cos\theta}}_{(CMS)} = 1 + \frac{2t}{s}$

we adopt as 2 variables s and the cos(θ) in the CM frame.

Remember:

$$P_J(z) = \frac{1}{2^d j!}\frac{d^d}{dz^j}(z^2-1)^d \quad w/ \quad \int_{-1}^{1} dz\, P_J(z) P_K(z) = \frac{2}{2j+1}\delta_{jK}$$

Legendre Polinomial (ortonormal) used to rewrite $\mathcal{M}$

basically I wrote the amplitude as a sum over fixed angular momentum amplitudes

and we can always decompose $\mathcal{M}$:

$$\mathcal{M}(s,t) = 16\pi \sum_j (2j+1)\, a_j(\underline{\underline{s}})\, P_j(\underline{\underline{\cos\theta}})$$

We write $\mathcal{M}$ as orthonormal decomposition in polynomials that allows to factorize the s, t dependence

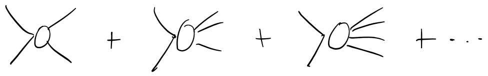
Whatever amplitude can be rewritten as a sum of partial waves times the Legendre polinomials

$$a_j(s) = \frac{1}{32\pi}\int_{-1}^{+1} d\cos\theta\, P_J(\cos\theta)\, \mathcal{M}(s\cos\theta)$$

Now we apply this relation to the scattering (□) to the partial waves

Let's start with RH of the formula (*)

(Im □ with $\sum_x$ we should sum over all final states



dominant

We assume this diagram is dominant just for the seak of organization

we look at the case $i = f = x = 2$

$$\underbrace{\int \frac{d^3\vec{P}_3}{(2\pi)^3 2E_3}\frac{d^3\vec{P}_4}{(2\pi)^3 2E_4}}_{\text{Phase Space}} (2\pi)^4 \delta^4(P_i - P_f)\left(\mathcal{M}(s,t)\right)^2 =$$

Counting variables

$3+3-4=2$ variable of integration

$$d\Omega = d\cos\theta\, \frac{d\varphi}{2\pi}$$

In general
For n particles

$$P_s = \prod_{i=1}^{n} \frac{d^3\vec{P}_i}{(2\pi)^3 2E_i}(2\pi)^4\delta^4(\Sigma P_i)$$

we only have the direction of one of them free, everything else is fixed

WHY WE HAVE ONLY ONE INTEGRATING VARIABLE?

$$= \frac{1}{16\pi}\int_{-1}^{+1} d\cos\theta \underbrace{\left[16\pi \sum_j (2j+1)\, a_j(s)\, P_j(\cos\theta)\right]}_{\mathcal{M}} \underbrace{\left[16\pi \sum_k (2k+1)\, a_k^*(s)\, P_k(\cos\theta)\right]}_{\mathcal{M}^*} =$$

⤳ Now integrate over dcosθ: we use the orthonormal condition for this integration

$$= 32\pi \sum_j (2j+1)\,|a_j(s)|^2$$

the angular dependency is gone we just have the energy dependency

This quantity doesn't depend on t but only on s

⤳ Remember that we are still assuming that the only final states are 2 particles

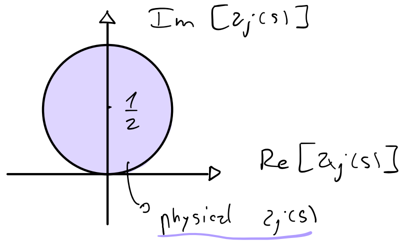
Recovering the LH side using cosθ = 1 and knowing $\mathrm{Im}(z) = z - z^*$

$$2\,\mathrm{Im}\,\mathcal{M}(s,\overset{\cos\theta=1}{0}) = 32\pi \sum_j (2j+1)\,\mathrm{Im}[a_j(s)]\,\overbrace{P_j(1)}^{=1\ \forall j}$$

⇒ That has to be valid for any j ⇒ $\boxed{\mathrm{Im}[a_j(s)] = |a_j(s)|^2}$

Since a_j complex $a_j(s) = x + iy$ we have equation of a circle: $y = x^2 + y^2$

UNITARITY CIRCLE



⇒ If that was an equality then a_j value lies on the circumference

but it's not in reality... we consider just 2 final states...

we have to add terms on the Im side

⇒ $y > x^2 + y^2$

Basically $\mathcal{M} - \mathcal{M}^* = i \sum_x \int(\quad | + \dots$

accounting for all the terms it turns out a_j lives in the circle $|a_j(s)| \leq 1$

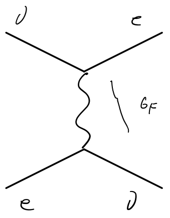
$|\mathrm{Re}\,(a_j(s))| \leq \frac{1}{2}$

⇒ Every partial amplitude has to be smaller than 1 in module, independently of the energy. It doesn't matter the energy, it cannot be greater than 1.

So the amplitude has to be $a_j \sim$ const or $1/s^n$.
It cannot go as $\sqrt{s}, s, s^{3/2}$ otherwise with increasing energy it will pass 1 at some point

<u>Example:</u> Fermi Theory

In the IR it goes to zero so no problems



$[M(2\to2)] = 1$ a $2\to2$ scattering has no dimension

Proof:

Phase space 2 particle

$$\sigma = \frac{1}{2s} |\mathcal{M}|^2 d\phi_2$$

↳ adimensional

$$\left[\frac{d^3p}{(2\pi)^3 2E} \frac{d^3p}{(2\pi)^3 2E} \delta^4(p) \right] = 0$$

$3-1=2$ $\quad 3-1=2 \quad -4$

↳ $\int d^4p\, \delta^4(p) = 1$

In the Fermi theory we have $\mathcal{M} \propto G_F \cdot s$

$\frac{1}{M^2}$

↳ has to be M^2 because $[\mathcal{M}] = 1$

Hence this amplitude grows with energy $\mathcal{M} = G_F \cdot s \Rightarrow a_0$ (since does not depend on θ)

From unitarity $\Rightarrow$ I know $|G_F \cdot s| < 1 \rightarrow \sqrt{s} < \frac{1}{\sqrt{G_F}} \sim v$

The theory is valid till here.
~ 300 GeV

$\Rightarrow$ Generically EFT violates UNITARITY.

8) Illustrate through QFT examples, matching at tree level and/or at one loop

The classic example of EFT theory of low-energy weak interactions is Fermi Theory.

The Full UV-Theory is the FULL SM, and we can match onto EFT by transitioning to a theory valid at momenta small compared to $M_{W,Z}$.

Since the weak interactions are perturbative, the matching can be done order by order in perturbation theory.

SM Theory

The W boson interacts with quarks and leptons via the weak current:

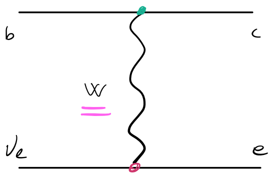
$$J_W^{\mu} = \underset{\text{CKM}}{V_{ij}} (\bar{u}_i \gamma^{\mu} P_L d_j) + (\bar{\nu}_e \gamma^{\mu} P_L \ell)$$

- u : UP-TYPE
- d : DOWN-TYPE

↳ NO MIXING MATRIX

⤳ Using neutrino Flavour eigenstates

The TREE-LEVEL amplitude for semileptonic b → c decay is:



$$\mathcal{A} = \left(\frac{-ig}{\sqrt{2}}\right) \underline{V_{cb}}\, \underline{(\bar{c}\gamma^{\mu} P_L b)}\underline{(\bar{\ell}\gamma^{\nu} P_L \nu_e)} \underline{\left(\frac{-ig_{\mu\nu}}{p^2 - M_W^2}\right)}$$

$-i\Gamma M_W$

where $g/\sqrt{2}$ is the W coupling constant. For low momentum transfer $p \ll M_W$ we can expand the W propagator

$$\frac{1}{p^2 - M_W^2} = -\frac{1}{M_W^2}\left(1 + \frac{p^2}{M_W^2} + \frac{p^4}{M_W^4} + \dots\right)$$

giving different orders in the EFT expansion parameter p/M_W.

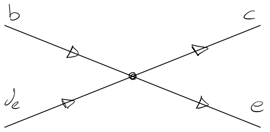
Taking only the first term we'll have

$$\mathcal{A} = \frac{i}{M_W^2}\left(\frac{-ig}{\sqrt{2}}\right) V_{cb} (\bar{c}\gamma^{\mu} P_L b)(\bar{\ell}\gamma_{\mu} P_L \nu_e) + \mathcal{O}\left(\frac{1}{M_W^4}\right)$$

Now the EFT Theory

$$\mathcal{L} = -\frac{g^2}{2M_W^2} V_{cb} (\bar{c}\gamma^\mu P_L b)(\bar{\ell}\gamma_\mu P_L \nu_e) + O\left(\frac{1}{M_W^4}\right)$$

this is the lowest order Lagrangian for semileptonic b → c decay in the EFT and is represented by the vertex :



with

$$\boxed{\frac{G_F}{\sqrt{2}} \equiv \frac{g^2}{8M_W^2} = \frac{1}{2v^2}}$$

MATCHING

where $v \sim 246$ GeV is the scale of electroweak symmetry breaking.

$\hookrightarrow$ $$\frac{G_F}{\sqrt{2}} = \frac{\cancel{g^2}}{8\left(\frac{\cancel{g^2} v^2}{4}\right)} = \frac{1}{2v^2}$$

$\Rightarrow$ g cancels

$$M_W^2\big|_{SM} = \frac{g^2 v^2}{4}$$

we have

$$\mathcal{L} = -\frac{4G_F}{\sqrt{2}} V_{cb} (\bar{c}\gamma^\mu P_L b)(\bar{\ell}\gamma^\mu P_L \nu_e)$$

How do you measure G_F? Muon decay $\frac{1}{\tau_\mu} = \Gamma_\mu = \frac{G_F^2 m_\mu^5}{192\pi^3}$

This is the low - energy limit of the the SM.

The EFT has dynamical W bosons and the effect of W exchange in the SM has been included via dimension -6 4 fermion operators.

$\rightsquigarrow$ This is called " integrating out " a heavy particle, the W boson

MATCHING AT TREE-LEVEL

The starting point is the UV theory which consists of

$$\mathcal{L} = \underbrace{\bar{\psi} i \not{\partial} \psi}_{\substack{\text{Light Fermion} \\ m \ll M \\ \text{therefore set to } 0}} + \underbrace{\frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2} M^2 \phi^2}_{\text{Heavy Scalar}} - \underbrace{\lambda \phi \bar{\psi} \psi}_{\text{Yukawa}}$$

- ψ : light Fermion
- ϕ : Heavy scalar Field with mass M
- λ : Coupling constant between ψ and ϕ

$\longrightarrow$ $m_f \ll E \ll M$
$\quad \hookrightarrow$ energy of experiment

if $m_f = 0$ we hav $\delta = \dfrac{E}{M}$

We want to construct an effective theory $\mathcal{L}_{EFT} = \mathcal{L}(\psi)$ (NO ϕ)

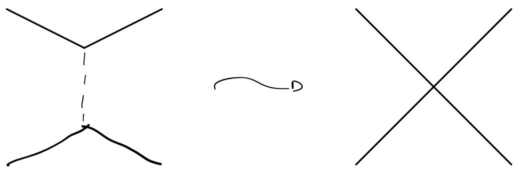
$\Rightarrow$ INTEGRATING OUT ϕ

Since the scalar ϕ has a large mass ($M > E$, where E is the energy scale of intrest) we can integrate it out and obtain an effective 4-Fermion interaction.

What we will do:

1) Let's compute $f\bar{f} \to f\bar{f}$ in the Full Theory and EFT

2) Match them



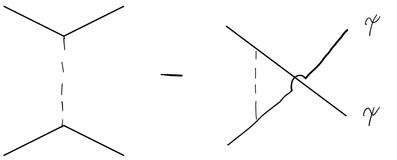
propagator reduces to a point

Full Theory

In the Full theory

$$i\mathcal{M} = \bar{u}_3 u_1 (-i\lambda)(-i\lambda) \bar{u}_4 u_2 \frac{i}{(p_3 - p_2)^2 - M^2} - (3 \leftrightarrow 4)$$

and the diagrams will be



− sign comes from exchanging of 2 Fermions (QFT)

So

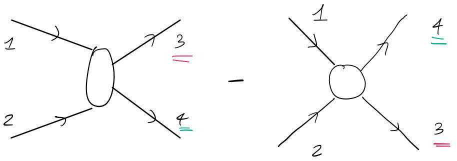
$$\mathcal{L}_{EFT} = \bar{\psi}(i\not{\partial})\psi + \frac{C_6}{2M^2}(\bar{\psi}\psi)(\bar{\psi}\psi)$$

↳ convenience

C : we need to find it

"Integrating out ϕ"

EFT



$$i\mathcal{M} = \mu_s \, i \frac{C_6}{M^2} \qquad \text{with} \qquad \mu_s = \bar{u}_3 u_1 \bar{u}_4 u_2 - (3 \leftrightarrow 4)$$

we match them and we obtain $C_6 = \lambda^2$

What about the next term?

$$\frac{1}{(p_3 - p_1)^2 - M^2} = -\frac{1}{M^2}\left[1 + \frac{(p_1 - p_3)^2}{M^2} + \dots\right]$$

DERIVATIVE EXPANSION

That operator describes this term (only for this was the matching)

↳ For this we need another operator which is dim-8

$$\mathcal{L}_{EFT} = \bar{\psi}(i\not{\partial})\psi + \frac{C_6}{2M^2}(\bar{\psi}\psi)(\bar{\psi}\psi) + \frac{C_8}{M^4}\underbrace{(\partial_\mu\bar{\psi}\,\partial^\mu\psi)(\bar{\psi}\psi)}_{\text{dim 8}}$$

We can check that this operator generates an amplitude term:

$$\mu_s\left(\frac{-iC_8}{M^4}\right)(p_1\cdot p_3 + p_2\cdot p_4)$$

We always have 4 fermions we add derivatives which pop out the terms in the propagation above

$$\leadsto \quad \underline{\underline{C_8 = -\lambda^2}}$$

Matching with EQUATIONS OF MOTION

We write EQ. of motion for ϕ $$\frac{\partial\mathcal{L}}{\partial\phi} - \partial_\mu\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)} = 0$$

$$\Rightarrow \quad (-M^2\phi - \Box\phi) = \lambda\psi\bar\psi$$

$$(M^2+\Box)\phi = -\lambda\bar\psi\chi$$

Extract ϕ:

$$\phi = \frac{-\lambda\bar\psi\chi}{(M^2+\Box)} = \frac{-\lambda\bar\psi\chi}{M^2\left(1+\frac{\Box}{M^2}\right)} = -\left[1 - \frac{\Box}{M^2} + \frac{\Box^2}{M^4} - \dots\right]\frac{\lambda}{M^2}\bar\psi\chi$$

Now put it inside in the UV lagrangian

$$\bar\psi i\not{\partial}\psi - \frac{1}{2}\left(\partial_\mu[\quad]\frac{\lambda}{M^2}\bar\psi\chi\right)^2 - \frac{1}{2}M^2\left(-[\quad]\frac{\lambda}{M^2}\bar\psi\chi\right)^2 - \lambda\frac{\lambda}{M^2}\cdot[\quad]\bar\psi\chi\bar\psi\chi$$

$\downarrow$ taking 1 in [ ] $\downarrow$

$$-\frac{1}{2}\frac{\lambda^2}{M^2}\bar\psi\chi\bar\psi\chi \quad + \quad \frac{\lambda}{M^2}\bar\psi\chi\bar\psi\chi$$

so same term as before $\leftarrow$ $$= \left(-\frac{1}{2}+1\right)\frac{\lambda^2}{M^2}\bar\psi\chi\bar\psi\chi$$

MATCHING

9| Discuss through examples the role of renormalization, logs and RGEs in QFT and EFT.

What is a Renormalizeable Theory? ⟹ You can compute scattering amplitudes at any order in PERTURBATION THEORY at all the time at different order I get infinities that are reabsorbed in the definitions of the constants

⤳ REABSORBE THE DIVERGENCES IN THE AVAILEABLE PARAMETERS

Renormalizeability: you don't need to modify the Lagrangian

To define a model in QFT you have to add all terms that are respecting the symmetries otherwise the perturbative approach will generate them.

Question: Does an EFT by construction satisfy the definition of the RENORMALIZEABILITY?

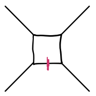
NO! but is misleading

It's different: in the end we will understand that we just have to REDEFINE the RENORMALEAZABILITY and we will understand that it's as RENORMALIZEABLE AS the UV theory.

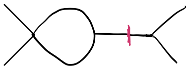
DEFINITIONS:

1| Particle Irreducible diagram (1PI)

Example:



- If I cut the loop goes away 1PI



- The loop is still there NON 1PI

2| Superficial DEGREE OF DIVERGENCE D

$$\int d^4K \, K^i \rightarrow D = 4 + i$$

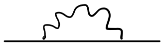
The integral of this diverges when $4 \geq -i$

$$\int_a^\infty dx \begin{cases} x^n & n \geq 0 \\ 1/x & \\ 1/x^n & n \geq 2 \end{cases}$$

Example: QED is renormalizeable

Has to do with the normalization of

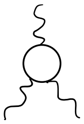
You take all the 2-points



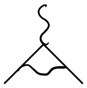
$\longrightarrow Z_2 \sim \psi$

$\longrightarrow Z_3 \sim A_\mu$

all the 3-points



$\longrightarrow$ zero $\rightsquigarrow$ Fury's Theorem



$\longrightarrow Z_1 \sim e$

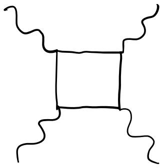
There are also RENORMALIZATION CONDITIONS :

$$\underbrace{\begin{aligned} &\Pi(0) = 0 && \rightsquigarrow m_\gamma = 0 \\ &\Gamma^\mu(0) = \gamma^\mu && \rightsquigarrow e \text{ at } Q^2 = 0 \text{ is } \sqrt{\alpha} \\ &\Sigma(mp) = 0 && \rightsquigarrow m \\ &\Sigma'(mp) = 0 && \rightsquigarrow \psi \end{aligned}}$$

With 2 measurements QED is set

?

Let's compute



$$\int d^4k \, \frac{1}{k^4}$$

4 propagators

cause propagator of fermions is

$$\frac{1}{\not{k} - m}$$

Since $\displaystyle \frac{d^4k}{k^4} = \frac{k^3}{k^4} \overbrace{\frac{dk}{k}}^{\log} d\Omega = \int d^4k \, \frac{1}{k^4} \sim \log \Lambda$

BECAUSE IT'S ADIMENTIONAL

Should be divergent!

Is this really divergent?

This is adimentional ~> The only thing in the log can be just g

$$\sim \log \Lambda \, (g^{\mu\nu} g^{\rho\sigma} + \dots)$$

and I see that IF I contract any amplitude contracted with the EXTERNAL momentum of the photon

$$p^{\mu} \mathcal{M}_{\mu\nu\rho\sigma} = 0$$

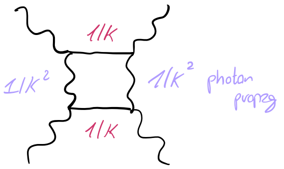
→ BUT with this structure we have a result ≠ 0 !

PROBLEM

Has to be zero otherwise the term violates gauge invar.

TERM FORBIDDEN BY GAUGE INVARIANCE

We can also have other loops:



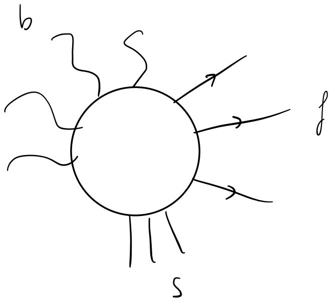
$$\sim \int \frac{1}{k^6} d^4k$$

FINITE

FROM 4-POINT ON in QED there are no more divergences because I reabsorb them in m and e.

This can be generalized

$$\boxed{D_{f,b,s} = 4 - \frac{3}{2} f - b - s}$$



This can be most easily derived via dimensional analysis

~ Remembering that the amplitude is Four Transf of the Green Function

$$\underbrace{(2\pi)^4\delta^4(\Sigma p)}_{\substack{-4 \\ \text{cons. momentum}}}\mathcal{M} \sim \prod_{i=1}^{s,b}\int \overset{-4}{d^4x_i}\, e^{\pm i p_i \cdot x}\,\Box_i \quad -2 \qquad \begin{matrix}\text{integ} & \text{deriv} \\ -4 & +2 = -2\end{matrix}$$

$$\prod_{j=1}^{F}\int \overset{-4}{d^4y_j}\, e^{\pm i q y_i}\,\not{\partial}_j \quad -3 \qquad \begin{matrix}\text{integ} & \text{deriv} \\ -4 & +1 = -3\end{matrix}$$

$$\langle 0|\overset{3/2\,f}{\underbrace{\psi\dots\psi}_{f}}\;\overset{1\times b}{\underbrace{A_1\dots A_b}_{b}}\;\overset{1\times s}{\underbrace{\phi_1\dots\phi_n}_{s}}|0\rangle$$

Let's compute the dimension of the generic scattering amplitude knowing

$$[\psi] = 3/2\,, \quad [A] = [\phi] = 1$$

$$[\mathcal{M}] = \underbrace{4 - 3F - 2b - 2s}_{\text{derivative}} \underbrace{+ \frac{3}{2}F + b + s}_{\substack{\text{Green Function} \\ \langle | \cdot | \rangle}} = \boxed{4 - \frac{3}{2}f - b - s}$$

$\rightarrow$ Superficial degree of D goes with the dimension of my amplitude.

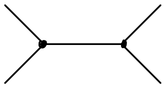
Let's now look at a theory with higher dimensional operators

$$\mathcal{L} = -\frac{1}{2}\phi(\Box + m^2)\phi + g_1\phi^3 + g_2\phi^2\Box\phi^3 + \dots$$

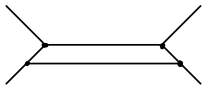
Dimension couplings g $\qquad [g_1] = \Delta_1 = 1 \qquad \Delta_2 = -3 \qquad \dots \quad g_i \rightarrow \Delta_i$

Let's consider a diagram with n_i insertions

This has $n_1 = 2$



This one has $n_1 = 4$



$i = 1$

Now we can write the dimension of a generic diagram. I can say that considering A LOOP with n_i insertion of the vertex g_i with Δ_i

Generic degree of divergence at a loop with dimensionless couplings

$$\int K^D \xrightarrow[EFT]{in} \prod_i g_i^{n_i} \int K^{D-\sum n_i \Delta_i}$$

IN THE END

$$D_{F,b,s,n_i} = 4 - \tfrac{3}{2}F - b - s \underbrace{- \textstyle\sum_i n_i \Delta_i}_{\text{new in EFT}}$$

new in EFT
because is DUE TO THE EFT coupling
⇓
Huge! EFT couplings change the dimension D

≫ In EFT I have to count the dimension of interactions

⟶ In QED e is dimentionless and I don't count.

IN EFT couplings change dimension of the amplitude.

Not surprisingly :

IF interactions have $\Delta_i < 0$ then there will be an infinite number of values n_i and F and b with $D_{F,b,s,n_i} > 0$

Interaction

$\Delta_i = 0$ ⟶ NOTHING HAPPENS

$\Delta_i < 0$ ⟶ IRRELEVANT ⇒ increases D ∼ more divergt ⟶ less renormalizeable

$\Delta_i > 0$ ⟶ RELEVANT ⇒ decreases D ∼ less divergt ⟶ MORE renormalizeable

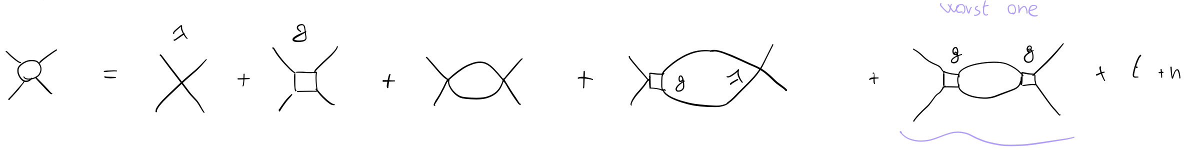
NOTE THAT : a theory might be renormalizeable due to behaviour of propagators (SSB)

NON RENORMALIZEABLE THEORIES are still "Renormalizeable" ie they are order by order in the dimension.

Let's do a real example

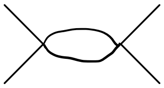
$$\mathcal{L} = -\frac{1}{2}\phi(\Box + m^2)\phi + \frac{g}{4!}\underbrace{\phi^2\Box\phi^2}_{dim 6} + \frac{\lambda}{4!}\phi^4 \qquad \Delta_g = 2 \quad , \Delta_\lambda = 0$$

Scattering 4 Fields:



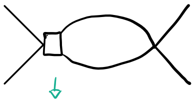
Feynm Rule: $-i\lambda$ $\quad -igp_i^2$

Now



$$= \lambda^2 \int \frac{d^4K}{K^4} \sim \lambda^2 \log \Lambda \xrightarrow[\text{ABSORB } \log]{\text{REDEFINITION OF } \lambda \text{ TO } \lambda_R} \lambda_R \delta_\lambda \sim \lambda^2$$

✓ RENORMALIZEABL

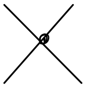


$$= g\lambda \int \frac{d^4K}{K^4} \underline{f(\overset{\text{LOOP}}{K^2}, p^2)} \xrightarrow{\text{RED}} g\lambda\,(\underbrace{b_2 \Lambda^2}_{\text{worst! QUADRATIC dependence}} + b_1 p^2 \log \Lambda)$$

$$= g\lambda \int \frac{k^3 dk\, d\Omega}{k^4} \cdot f(k^2, p^2) \rightsquigarrow \int \frac{1}{K} K^2 dK + \int \frac{1}{K} p^2 dK =$$

CAN I DEAL WITH THIS?

In the end not that bad case I'll have counterterms from



that are proportional to $g\lambda$

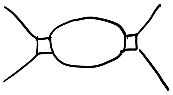
$-i\lambda$  1° $\lambda_R \delta_R + g_R \delta_g p^2$ 2° integral: Adding counter terms

$-igp^2$  2° $-g_R \lambda_R b_2 \Lambda^2 - g_R \lambda_R b_2$ 1° integral: RENORMAL OF λ

⇝ Up to here the model is RENORMALIZEABLE

PROBLEM

However if I add 2 higher-dim operators we see that things brake



$$= g^2 (c_1 \Lambda^4 + c_2 \Lambda^2 p^2 + \underbrace{c_3 p^4 \log \Lambda}_{\text{PROBLEM!}})$$

PROBLEM! NOT IN THE LAGRANGIAN

nothing scales as p^4 in the $\mathcal{L}$

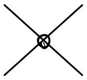
It seems that I could only renormalize $c_2 \Lambda^2 p^2$ as the tree-level behaviour.

We need to enlarge the set of operators

$$\mathcal{L} = -\frac{1}{2} \phi (\Box + m^2) \phi + \lambda_R Z_\lambda \phi^4 + g_R Z_g \phi^2 \Box \phi^2 + \overbrace{K_R Z_K \underbrace{\phi^2 \Box^2 \phi^2}_{\text{dim } 6}}^{\text{New}}$$

$Z_i = 1 + \delta_i$

Now it gives



$$\sim \underbrace{\lambda_R \delta_\lambda}_{\substack{\downarrow \\ -g_R^2 c_1 \Lambda^4}} + \underbrace{g_R \delta_g p^2}_{\substack{\downarrow \\ -g_R^2 c_2 \Lambda^2}} + \underbrace{K_R \delta_K}_{\substack{\downarrow \\ -g_R^2 c_3 \log \Lambda}} p^4$$

⇒ Now all infinities cancel.

This is why people say that EFT ARE NOT RENORMALIZEABLE: If you add any expom in the loop, the theory needs to be EXPANDED ↳ change!

This  has to be seen as loop correction of renormalizeable interaction.

Renormalization Group

The phrase renormalization group refers to the invariance of observable under changes in the way things are calculated

Those are two versions of the renormalization group used in QFT:

Wilsonian Renormaliz Group

In a finite theory with UV cutoff Λ, physics at energies $E \ll \Lambda$ is independent of the precise value of Λ. Changing Λ changes the couplings in the theory so that observables remain the same

Continuum Renormaliz Group

Observables are independent of the renormalization conditions, in particular, of the scales $\{p_0\}$ at which we choose to define our renormalized quantities. This invariance holds after the theory is renormalized and cutoff is removed ($\Lambda = \infty$, $d = 4$). In dim. regularizations with $\overline{MS}$ scheme, the scales $\{p_0\}$ are replaced by μ, and the continuum renormalization group comes from μ independence

The 2 approches are similar, however in both cases the fact that Theory is independent of something means that one can set up differential equations such as

$$\frac{d}{d\Lambda} X = 0 \,, \quad \frac{d}{dP_0} X = 0 \quad \text{or} \quad \frac{dX}{d\mu} = 0$$

w/ X some observable

$\rightsquigarrow$ Solving those leads to a trajectory in the space of theories.

RGE means flow along these trajectories. We look at

- COUPLING CONSTANTS
- OPERATORS
- GREEN'S FUNCTION

Preview of RGE:

- We know $e^2_{eff}(Q) = \dfrac{e_R^2}{1 - \dfrac{e_R^2}{12\pi^2} \ln \dfrac{Q^2}{m^2}}$ RUNNING COUPLING this equation can be written as $$\frac{1}{e^2_{eff}(Q)} = \frac{1}{e_R^2(\mu)} - \frac{1}{12\pi^2}\ln\frac{Q^2}{m^2}$$

↳ Independent of μ, derivative

$$0 = -\frac{2}{e^3_{eff}}\frac{d}{d\mu}e_{eff} + \frac{1}{12\pi^2}\frac{2}{\mu} \Rightarrow \boxed{\overbrace{\mu\frac{de_{eff}}{d\mu}}^{\beta\text{-function}} = \frac{e^3_{eff}}{12\pi^2}}$$

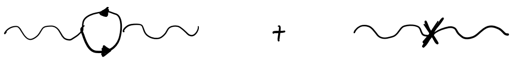
RENORMALIZATION
GROUP
EQUATION

Running Couplings

Example: Running of $e_{eff}(\mu)$ - scale dependent electric charge

VACUUM POLARIZATION DIAGRAMS

calculation of loop + the counterterm



$$+ \quad = -i\left(p^2 g^{\mu\nu} - p^\mu p^\nu\right)\left(\underline{\underline{e_R^2 \Pi_2(p^2)}} + \underline{\underline{\delta_3}}\right)$$

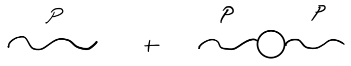
COUNTERTERM

$$\to \Pi_2(p^2) = \frac{1}{2\pi^2}\int_0^1 dx\,(1-x)x\left[\frac{2}{\varepsilon} + \log\left(\frac{\tilde{\mu}^2}{m_R^2 - p^2 x(1-x)}\right)\right]$$

In dimensional regularization $d = 4 - \varepsilon$

So the dressed photon propagator is

$$i\,G^{\mu\nu} =$$



$$= -i\,\frac{\left[1 - e^2\Pi_2(p^2)\right]}{p^2}\,g^{\mu\nu} + p^\mu p^\nu$$

By embedding this off-shell amplitude into a scattering diagram we extracted an effective Coulomb potential whose Fourier Transform was

$$\tilde{V}(p^2) = e_R^2\,\frac{1 - e_R^2\,\Pi_2(p^2)}{p^2}$$

Defining the gauge coupling e_R so that $\tilde{V}(p_0^2) = \frac{e_R^2}{p_0^2}$ write the renorm. V

@ $p_0 \to \delta_3$ since $\Pi_2(p_0^2)$ is infinite

SAME AS P_0 FIXES δ_3

An the potential at another p scale is

$$p^2\,\tilde{V}(p) = e^2 - e^4\,\Pi_2(p^2) + \ldots = e_R^2 - e_R^4\left[\Pi_2(p^2) - \Pi_2(p_0^2)\right] + \ldots$$

Hence

$$\tilde{V}(p^2) = \frac{e_R^2}{p^2}\left\{1 + \frac{e_R^2}{2\pi^2}\int_0^1 dx\; x(1-x)\ln\left(\frac{p^2 x(1-x) - m^2}{p_0^2 x(1-x) - m^2}\right) + \mathcal{O}(e_R^4)\right\}$$

Which is FINITE and ε and μ independent, but depends on p_0.

CASE $p \gg m$

$$\tilde{V}(p^2) = \frac{e_R^2}{p^2}\left(1 + \frac{e_R^2}{12\pi^2}\,\underline{\log}\,\frac{p^2}{p_0}\right)$$

LARGE LOG that RG will solve

⟹ When this logarithm is this large appears at all orders.
(One can imagine that, since $e_R^2/12\pi \sim 10^{-3}$ then at higher orders log will vanish but there exist a scale when $p^2 \gg p_0^2$ where $\log p/p_0 \sim 10^3$ and so $\left(\frac{e_R}{12\pi}\right)\left(\log\frac{p}{p_0}\right) \sim$ ORDER 1.)

We can include to the propagator all the possible terms

$iG^{\mu\nu} =$ 

And they translate into corrections to the potential (1PI-loop contributions)

$$\hat{V}(p^2) = \frac{e_R^2}{p^2}\left[1 + \frac{e_R^2}{12\pi}\ln\frac{p^2}{p_0^2} + \left(\frac{e_R^2}{12\pi^2}\ln\frac{p}{p_0}\right)^2 + \dots\right] = \frac{1}{p^2}\left[\frac{e_R^2}{1 - \frac{e_R^2}{12\pi^2}\ln\frac{p^2}{p_0^2}}\right]$$

We then define the effective coupling through the potential $e_{eff}^2(p^2) \equiv p^2\hat{V}(p^2)$ so

$$e_{eff}^2(p^2) = \frac{e_R^2}{1 - \frac{e_R^2}{12\pi^2}\ln\frac{p^2}{p_0^2}}$$

LEADING LOGARITHMIC SUMMATION

$e_{eff}^2(p^0) = e_R$

Once all the 1PI-loop contributions are included, the next terms we are missing should be subleading in some expansion.

TERMS INCLUDED IN THE EFFECTIVE CHARGE ARE OF THE FORM $e_R^2\left(e_R^2\ln\frac{p^2}{p_0^2}\right)^n$ $n \geq 0$

2-LOOP 1PI CONTRIBUTIONS TO BE SUBLEADING THEY SHOULD BE THE FORM $e_R^4\left(e_R^2\ln\frac{p^2}{p_0^2}\right)^n$

^D To check we would need to perform the full $O(e_R^4)$ calculation ⟹
to understand that there are not LL from a 1PI 2-loop calculation

RG gives shortcut to systematic RESUMMATION beyond LEADING LOGARITHMIC

- If there were no large logs we would be fine with fixed-order perturbation theory
- If there are large logs $\Rightarrow$ NEED RG

The RGE (Renormalization Group Equation) then comes from requiring that the potential is independent of p_0^2

$$p_0^2 \frac{d}{dp_0^2} \tilde{V}(p^2) = 0$$

- $\tilde{V}(p^2)$ has EXPLICIT p_0^2 dependence
  IMPLICIT p_0^2 dependence through the scale where e_R is defined

Infact recall e_R was defined so that $p_0^2 \tilde{V}(p_0^2) = e_R^2$ exactly, and the effective charge is defined by $e_{eff}^2(p^2) \equiv p^2 \tilde{V}(p^2)$ we can make the p_0^2 dependence of $\tilde{V}(p^2)$ explicit by replacing e_R by $e_{eff}(p_0^2)$

So

$$\tilde{V}(p^2) = \frac{e_{eff}^2}{p^2} \left(1 - \frac{e_{eff}^2}{12\pi^2} \ln \frac{p_0^2}{p^2} \right) + \dots$$

then at 1-loop the RGE is

$$0 = p_0^2 \frac{d}{dp_0^2} \tilde{V}(p^2) = \frac{1}{p^2} \left(p_0^2 \frac{de_{eff}}{dp_0^2} 2 e_{eff} - \frac{e_{eff}^4}{12\pi^2} - p_0^2 \frac{de_{eff}}{dp_0^2} \frac{e_{eff}^3}{3\pi^2} \ln \frac{p_0^2}{p^2} + \dots \right)$$

to solve this in perturbation theory we note that de_{eff}/dp_0^2 must scale as e_{eff}^3 and so the third term inside the bracket is subleading.

Thus 1-loop RGE is

$$p_0^2 \frac{de_{eff}}{dp_0^2} = \frac{e_{eff}^3}{24\pi^2}$$

Solving this differential equation with boundary condition $e_{eff}(p_0) = e_R$ gives

$$e_{eff}^2(p^2) = \frac{e_R^2}{1 - \frac{e_R^2}{12\pi^2} \ln \frac{p^2}{p_0^2}} \Bigg]$$

Same effective charge we got summing up 1PI diagrams

UNIVERSALITY OF LARGE LOGS

Large Logarithms arise when one scale is much bigger or much smaller than every relevant scale.

In the vacuum polarization calculation, we considered the limit where the off-shellness p^2 of the photon was much larger than the electron mass m^2.

In the limit where one scale is much larger than all the other scales, we can set all the other physical scales to zero to first approximation.

If we do this in the vacuum polarization diagram we find that the full vacuum polarization function at order e_R^2 is

$$\Pi(p^2) = \frac{e_R^2}{12\pi^2}\left[\frac{2}{\varepsilon} + \ln\left(\frac{\mu^2}{-p^2}\right) + \text{const}\right] + \delta_3 \qquad (\text{DR}) \qquad (*)$$

The equivalent result using a regulator with a dimensional UV cutoff is

$$\Pi(p^2) = \frac{e_R^2}{12\pi^2}\left[\ln\left(\frac{\Lambda^2}{-p^2}\right) + \text{const}\right] + \delta_3 \qquad (\text{PV})$$

If the only scale is p^2, the logarithm of p^2 must be compensated by a logarithm of some other unphysical scale, in this case the cutoff Λ^2 (or μ^2 in dimensional regularizations)

If we renormalize at SAME SCALE defining

$$\delta_3 = -\frac{e_R^2}{12\pi^2}\log\frac{\Lambda^2}{-p_0^2} \longrightarrow \Pi(p^2) = \frac{e_R^2}{12\pi}\left[\log\left(\frac{p_0^2}{p^2}\right) + \text{const}\right] \quad (\text{PV})$$

$$\delta_3 = \frac{e_R}{12\pi^2}\left(-\frac{1}{\varepsilon}\right) \longrightarrow \Pi(p^2) = \frac{e_R^2}{12\pi}\left[\log\left(\frac{\mu^2}{p^2}\right) + \text{const}\right] \quad (\text{DR})$$

However one can chose in dimensional regularization

$$\delta_3 = \frac{e_R^2}{12\pi^2}\left[-\frac{2}{\varepsilon} - \ln\frac{\mu^2}{p_0^2}\right] \longrightarrow \Pi(p^2) = \frac{e_R^2}{12\pi}\left[\log\left(\frac{p_0^2}{p^2}\right) + \text{const}\right]$$

that is equivalent of chosing $\mu^2 = p_0^2$ in $(*)$ and subtracting $1/\varepsilon$ term.

$\Longrightarrow$ μ RENORMALIZATION SCALE

Although we choose μ to be a physical scale, observable should be independent of μ. At fixed order in perturbation theory, verifying μ independence can be a strong theoretical cross check

Since μ is the renormalization point, the effective coupling becomes $e_{eff}(\mu)$ and the RGE becomes

$$\mu \frac{d e_{eff}(\mu)}{d\mu} = \frac{e_{eff}^{3}(\mu)}{12 \mu^{2}}$$

RENORMALIZATION GROUP FROM COUNTERTERMS

We have seen how large logs of the form $\ln Q_1^2/Q_2^2$ can be resummed though a differential equation which establishes that physical quantities are independent of the scale Q_0^2 where the renormalized coupling is defined

Let's recall where the factors μ come from. Recall the BARE Lagrangian for QED

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi}^{\circ} (i \not{\partial} - e^{\circ} \gamma^{\mu} A^{\circ}_{\mu} - m^{\circ}) \psi^{\circ}$$

The quantities appearing here are infinite, or if we are in $d = 4 - \varepsilon$ dimensions they are finite but scale as the inverse powers of ε

The dimensions can be read off from the Lagrangian

$$[A^{\circ}_{\mu}] = \frac{d-2}{2} \qquad [\psi^{\circ}] = \frac{d-1}{2} \qquad [m^{\circ}] = 1 \qquad [e^{\circ}] = \frac{4-d}{2}$$

In particular we would like the charge e_R we expand in to be a number, and the fields to have canonical normalization.

We therefore rescale by

$$A_{\mu} = \frac{1}{\sqrt{Z_3}} A^{\circ}_{\mu} \qquad \psi = \frac{1}{\sqrt{Z_2}} \psi^{\circ} \qquad m_R = \frac{1}{Z_m} m^{\circ} \qquad e_R = \frac{1}{Z_e} \mu^{\frac{d-4}{2}} e^{\circ}$$

Which leads to

$$\mathcal{L} = -\frac{1}{4} Z_3 F_{\mu\nu} F^{\mu\nu} + i Z_2 \bar{\psi} \not{\partial} \psi - m_R Z_2 Z_m \bar{\psi}\psi - \mu^{\frac{4-d}{2}} e_R Z_e Z_2 \sqrt{Z_3}\, \bar{\psi} \not{A} \psi$$

with e_R and the Z_x dimensionless, even in $d = 4 - \varepsilon$ dimensions

Recall also the the 1-loop $\overline{MS}$ counterterms

$$\delta_2 = \frac{e_R^2}{16\pi^2}\left[-\frac{2}{\varepsilon}\right] \qquad \delta_3 = \frac{e_R^2}{16\pi^2}\left[-\frac{8}{3\varepsilon}\right] \qquad \delta_m = \frac{e_R^2}{16\pi^2}\left[-\frac{6}{\varepsilon}\right] \qquad \delta_e = \frac{e_R^2}{16\pi^2}\left[\frac{4}{3\varepsilon}\right]$$

where each of these counterterms is defined by $Z_x = 1 + \delta_x$

Notice that, since there is μ dependence in the RENORM $\mathcal{L}$ but not in the BARE $\mathcal{L}$
we must have

$$0 = \mu \frac{d}{d\mu} e^0 = \mu \frac{d}{d\mu}\left[\mu^{\frac{\varepsilon}{2}} e_R Z_e\right] = \mu^{\frac{\varepsilon}{2}} e_R Z_e \left[\frac{\varepsilon}{2} + \frac{\mu}{e_R}\frac{d}{d\mu} e_R + \frac{\mu}{Z_e}\frac{dZ_e}{d\mu}\right]$$

at leading order in e_R, $Z_e = 1$ and so

$$\mu \frac{d}{d\mu} e_R = -\frac{\varepsilon}{2} e_R$$

at next order, we have

$$\mu \frac{d}{d\mu} Z_e = \mu \frac{d}{d\mu}\left(1 + \frac{e_R^2}{16\pi^2}\frac{4}{3\varepsilon}\right) = \frac{1}{\varepsilon}\frac{e_R}{6\pi^2}\mu\frac{d}{d\mu} e_R = -\frac{e_R^2}{12\pi^2}$$

So

$$\beta(e_R) \equiv \mu \frac{d}{d\mu} e_R = -\frac{\varepsilon}{2} e_R + \frac{e_R^3}{12\pi^2}$$

In this theory is easy: $Z_e = \frac{Z_1}{Z_2\sqrt{Z_3}}$, $Z_1 = Z_2$, $Z_e = \frac{1}{\sqrt{Z_3}}$ QCD MORE COMPLEX

$\rightsquigarrow$ $\beta = \mu \frac{d\alpha}{d\mu}$ sometimes, with $\alpha = \frac{e_R^2}{4\pi}$

Follows

$$\beta(\alpha) = -2\alpha \left[\frac{\varepsilon}{2} + \left(\frac{\alpha}{4\pi}\right)\beta_0 + \left(\frac{\alpha}{4\pi}\right)^2 \beta_1 + \dots \right] \qquad \beta_0 = -\frac{4}{3} < 0$$

at leading order $\alpha(\mu) = \frac{2\pi}{\beta_0} \frac{1}{\log \frac{\mu}{\Lambda_{QED}}}$ (*)

$\mu < \Lambda_{QED}$

QED blows up at Λ_{QED} $\Rightarrow$ Landau Pole

- When $\alpha(m_e = 511\ \mathrm{keV}) = \frac{1}{137}$ we find $\Lambda_{QED} = 10^{286}$ eV

In writing solutions for RGE (*) we have swapped a number 1/137 for a dimensional scale Λ_{QED}.

This is known as DIMENSIONAL TRANSMUTATION.
We introduced the effective charge, we specified a scale and the value of α measured at that scale. But now we see that only a scale is needed.

This uncovers a very profound MISCONCEPTION of nature: electrodynamics is fundamentally not defined by the electric charge but by a dimensionful scale Λ_{QED}. But this has sense only refered to another scale: m_e.
So m_e/Λ_{QED} define QED completely.

While we have the counterterms handy, let us work out the RGE for the electron mass.
The bare mass m° must be independent of μ, so

$$0 = \mu \frac{d}{d\mu} m^\circ = \mu \frac{d}{d\mu} (Z_m m_R) = Z_m m_R \left[\frac{\mu}{m_R} \frac{d m_R}{d\mu} + \frac{\mu}{Z_m} \frac{d Z_m}{d\mu} \right]$$

We conventionally define

$$\gamma_m \equiv \frac{\mu}{m_R} \frac{d m_R}{d\mu}$$

γ_m ANOMALOUS DIMENSION

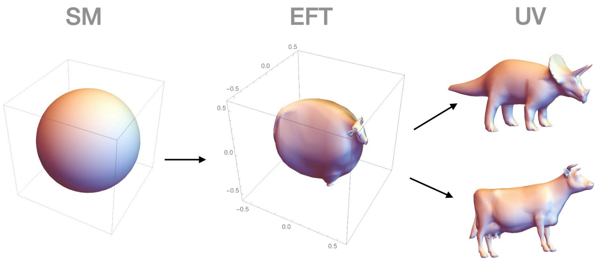
Since Z_m only depends on μ through e_R, we have

$$\gamma_m = -\frac{\mu}{Z_m}\frac{dZ_m}{d\mu} = -\frac{1}{Z_m}\frac{dZ_m}{de_R}\,\mu\frac{de_R}{d\mu}$$

At 1-loop, $Z_m = 1 - \frac{3e_R^2}{8\pi^2\varepsilon}$ and to leading non-vanishing order $\mu\frac{de_R}{d\mu} = \beta(e_R) = -\frac{\varepsilon}{2}e_R$

$$\gamma_m = -\frac{1}{1+\delta_m}\left(\frac{2}{e_R}\delta_m\right)\left(-\frac{\varepsilon}{2}e_R\right) = \delta_m\varepsilon = -\frac{3e_R^2}{8\pi^2} \qquad \text{FINITE.}$$

10) Consider SMEFT and present with details at least 2 aspects that you deem the most important ones (example: basis concept, RG running, global fits)



It's not a full theory ⟶ Add higher dim operators (start to shape towards the new complete theory) ⟶ UV Full theory

(SM symmetries are satisfied at low energy)

(I don't know which is the right one)

⟹ I study deviations from the SM at low energy, I probe at high Λ.
If i get closer, small Λ, higher energy, I understand which one is it.

Several thing to consider:

1. There are several possible higher dimentional operators that I can chose.

dim 5: only 1

$$\frac{1}{\Lambda}(\phi L_L)(\phi L_L)$$

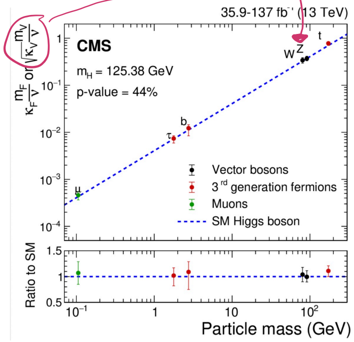
- mass neutrino Majorana
- VIOLATES Lepton Number

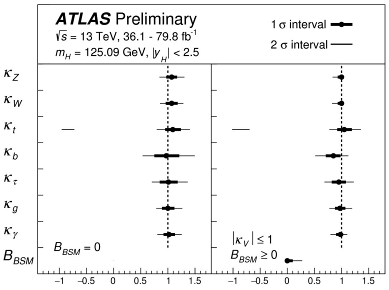
⤳ we know neutrino have masses

dim 6: several 2499 operators. How do we choose?

The SM success story

coupling ~→

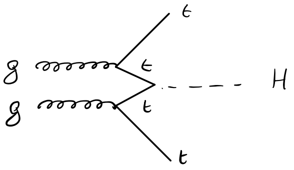


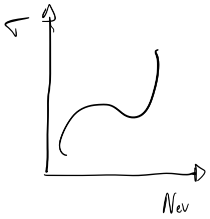


How do I measure the coupling of Higgs to particles?

USE PATRIAL WIDTH

~→ Not for the top: At LHC they use



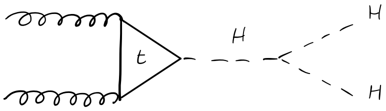


There is a MISSING PARTICLE: Higgs

Why? we know the mass but not the coupling.

To have the coupling I need to measure self higgs coupling.

⟹ DOUBLE HIGGS PRODUCTION



We can say that LHC after the Higgs didn't discover new particles and also there are no significant deviation from the expected values.

→ New resonances or effects have not been observed

NEW SEARCHES FOR BSM physics have pushed new physics to higher energy scales, motivating the use of indirect approaches like SMEFT

$\longrightarrow$ New physics is heavy : need for $\underbrace{\text{Precision Measurements + Heavy new Physics}}_{\text{SMEFT}}$

How do we search for new physics?

- is based on adding new higher dimensions operators
- EFT is generic and model independent

$\rightsquigarrow$ EFT serves as an interface between UV physics and low energy phenomena

Rules to follow:

- New physics sufficiently higher than all mass scales in the calcultions
- Global analysis : INCLUDE ALL POSSIBLE OPERATORS

Do you remember the number of dim 6 operators? There are 2499 BUT

- $\mathcal{L}$ is not a physical object
- There are link between operators
- It's possible to reduce to a min number of operators. (You chose a basis)

So now that we have 10 (for example) we write

$$\mathcal{L} = \mathcal{L}_{SM} + c_1 \mathcal{L}_1 + c_2 \mathcal{L}_2 + \dots$$

how do I measure $c_1, c_2 \dots$? Build observeables.

If we have 10 operators, how many observeable do I need?

This is phylosophy of SMEFT.

How many measurements do I need in the SM? 18

SMEFT

$$\boxed{\mathcal{L}_{eff} = \sum_i \frac{c_i O_i^D}{\Lambda^{D-4}}}$$

O_i : MUST RESPECT THE SM SYMMETRIES

Some are imposed, some are accidental

- gauge symmetry SU(3) × SU(2) × U(1)
- accidental symmetry

↳ gonna be broken in the SMEFT

dim 5 operator

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{1}{\Lambda}\mathcal{L}_5$$

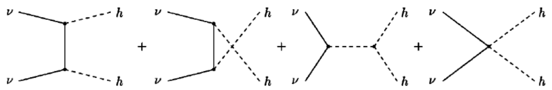
Majorana Neutrinos

There is only one such operator that can be added:

$$\mathcal{L} = \frac{c}{\Lambda}(L^T\varepsilon\phi)C(\phi^T\varepsilon L) + h.c \qquad \varepsilon \equiv i\sigma_2$$

When the higgs acquires a vev the term give rise to a Majorana neutrino mass $m_\nu = c\frac{v^2}{\Lambda}$

Computing all amplitudes



⟹ This grows with the energy = UNITARITY VIOLATIONS

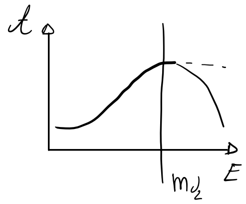
A UV completion of dim = 5 operator (there are few) is the see-saw model

$$\mathcal{L} = -y_0 \bar{L}\varepsilon\phi^* \nu_R - \frac{1}{2}M_R \nu_R^T C\nu_R + H.c.$$

with a Dirac Mass term and a Majorana one. ↳ We have a heavy neutrino and a light one

At this point the amplitude $\nu \to hh$ DOESN'T GROW anymore because the last interaction is NOT PRESENT anymore

Actually the amplitude $\mathcal{A}$ grows with energy until m_{ν_2} then it goes down

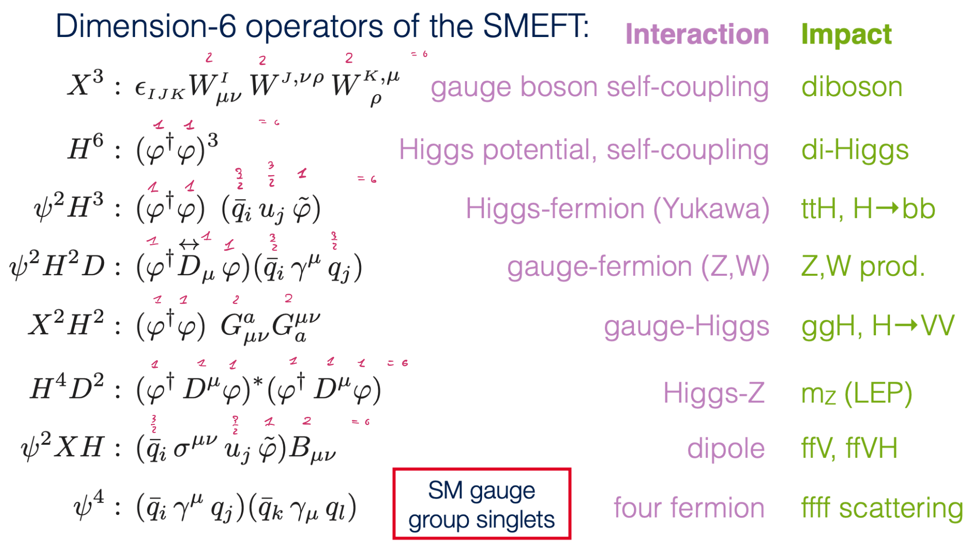


When does this amplitude violates unitarity? $1 > \frac{m_\nu E}{v^2}$ $E < \frac{v^2}{m_\nu} \sim 10^{15}$ GeV

By having a lower bound on neutrino we get an upper limit on Λ.
$\Rightarrow$ Nice example of the EFT Frame

Now we go back to dim 6.

The possible operators are



Dimension-6 operators of the SMEFT:	**Interaction**	**Impact**
$X^3 : \epsilon_{IJK}\overset{2}{W^I_{\mu\nu}}\,\overset{2}{W^{J,\nu\rho}}\,\overset{2}{W^{K,\mu}_{\rho}} \;{\scriptstyle =6}$	gauge boson self-coupling	diboson
$H^6 : (\overset{1}{\varphi}^\dagger \overset{1}{\varphi})^3 \;{\scriptstyle =6}$	Higgs potential, self-coupling	di-Higgs
$\psi^2 H^3 : (\overset{1}{\varphi}^\dagger \overset{1}{\varphi})\ (\overset{3/2}{\bar{q}_i}\, \overset{3/2}{u_j}\, \overset{1}{\tilde{\varphi}}) \;{\scriptstyle =6}$	Higgs-fermion (Yukawa)	ttH, H→bb
$\psi^2 H^2 D : (\overset{1}{\varphi}^\dagger \overset{1}{\overleftrightarrow{D}_\mu} \overset{1}{\varphi})(\overset{3/2}{\bar{q}_i}\, \gamma^\mu\, \overset{3/2}{q_j})$	gauge-fermion (Z,W)	Z,W prod.
$X^2 H^2 : (\overset{1}{\varphi}^\dagger \overset{1}{\varphi})\ \overset{2}{G^a_{\mu\nu}}\overset{2}{G_a^{\mu\nu}}$	gauge-Higgs	ggH, H→VV
$H^4 D^2 : (\overset{1}{\varphi}^\dagger\, \overset{1}{D^\mu} \overset{1}{\varphi})^*(\overset{1}{\varphi}^\dagger\, \overset{1}{D^\mu} \overset{1}{\varphi}) \;{\scriptstyle =6}$	Higgs-Z	m_Z (LEP)
$\psi^2 X H : (\overset{3/2}{\bar{q}_i}\, \sigma^{\mu\nu}\, \overset{3/2}{u_j}\, \overset{1}{\tilde{\varphi}})\overset{2}{B_{\mu\nu}} \;{\scriptstyle =6}$	dipole	ffV, ffVH
$\psi^4 : (\bar{q}_i\, \gamma^\mu\, q_j)(\bar{q}_k\, \gamma_\mu\, q_l)$	four fermion	ffff scattering

SM gauge group singlets

The First possible dim 6 operator is called GAUGE - BOSON - SELF - COUPLING

⤳ We put togheter 3 Field strenghts $\underbrace{W^{i}_{\mu\nu a}}_{\text{dim} 2} W^{j}_{\nu\rho b} W^{k}_{\rho\sigma c} \begin{cases} F^{abc} \\ \varepsilon^{ijk} \end{cases}$

$\partial_\mu A_\nu - \partial_\nu A_\mu$

⤳ The index is for the NON ABELIAN STRUCTURE

Algebra $[W^a, W^b] = i f^{abc} W^c$ SU(3)

$[\tau^i, \tau^j] = i \varepsilon^{ijk} \tau^k$ SU(2) Lee Algebra

---

Another operator is $\frac{C_i}{\Lambda^2} (\phi^\dagger \phi)^3$

- If you add this operator MODIFIES THE RELATION BETWEEN THE MASS and COUPLINGS

see below

Another important operator is

$\frac{(\phi^\dagger \phi)}{\Lambda^2} [C_i \underbrace{\bar{L} \phi e_R}_{\text{yukawa in dim 4}} + h.c.]$ ⤳ Yukawa operator

⤳ $\frac{y_F}{\sqrt{2}} = \frac{m_F}{v}$ → Measuring m_F and y_F they have to satisfy this relation ⇒ SM prediction

How do I see the effects of this operator?

So $\phi = \begin{pmatrix} 0 \\ \frac{v+h}{\sqrt{2}} \end{pmatrix}$ and $\phi^\dagger \phi = \frac{1}{2} (v+h)^2$

$$\frac{C_i}{\Lambda^2} \frac{1}{2} (v+h)^2 \underbrace{\boxed{\frac{1}{\sqrt{2}} (v+h) (\bar{e}_L e_R + h.c)}}_{\text{SM} (1/\sqrt{2})}$$

What is me? It's not the latter case we have also the other one

$$\left[\underline{\frac{C_i}{\Lambda^2} \frac{1}{2\sqrt{2}} (v+h)^3} + \underline{y_F \frac{1}{\sqrt{2}} (v+h)} \right] (\bar{e}_L e_R + h.c.)$$

$\rightsquigarrow \; m_F^{EFT} = \frac{C_i}{\Lambda^2} \frac{1}{2\sqrt{2}} v^3 + y_F \frac{1}{\sqrt{2}} v$ ① This is the mass in the EFT

$\rightsquigarrow \; y_F^{EFT} = \frac{C_i}{\Lambda^2} \frac{1}{2\sqrt{2}} 3 v^2 + \frac{y_F}{\sqrt{2}}$ ② This is the coupling to the Higgs

We can take C_i from above ① and substitute in ②

→ In DIM6 m_F and y_F^{EFT} are not related see above

The relation $\frac{y_F}{\sqrt{2}} = \frac{m_F}{v}$ is broken (doesn't hold anymore) and y_F, m_F are free parameters which depend on C_i

---

Looking to the Four Fermion operator $\bar{Q}_i \gamma^\mu Q_j \bar{Q}'_k \gamma_\mu Q'_l$

Flavour ⟹ increases the # operator

Only place in the SM where we have a Flavour dependence is in the charge currents (they are Flavour diagonal)
↳ $\bar{U}_i W^\mu V_{CKM} d_j$

Typical questions for exam is to check a random operator of these if is dim-6 and INVARIANT under SM gauge groups

The way of SMEFT

One can satisfy all the previous requirements by bulding an EFT on top of the SM that respects the gauge symmetries

$$\mathcal{L}_{SMEFT} = \mathcal{L}_{SM}^{(4)} + \frac{1}{\Lambda^2}\sum_i^{N_6} c_i O_i^{(6)} + \frac{1}{\Lambda^4}\sum_j c_j O_j^{(8)} + \dots$$

With the only assumption that all new states are heavier than energy probed by the experiment $\sqrt{s} < \Lambda$

The theory is renormalizeable order by order in $1/\Lambda$, perturbative computations can be consistently performed at any order, and the theory is predictive, i.e. well defined patterns of deviations are allowed, that can be further limited by adding assumptions from UV.

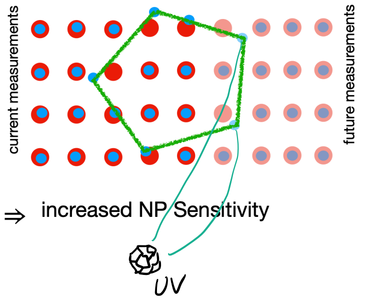
Operators can lead to larger effects at high energy (for different reasons)

A simple approach:

The master equation of an EFT approach has three key elements

$$\Delta Obs_n = \underbrace{Obs_n^{EXP}}_{\text{Most Precise experimental measurements}} - \underbrace{Obs_n^{SM}}_{\text{Most precise SM prediction (NLO, NNLO, ..)}} = \frac{1}{\Lambda^2}\underbrace{\sum_i a_{n,i}^{(6)}(\mu)}_{\text{Most precise EFT predictions}}\overbrace{C_i^{(6)}(\mu)}^{\text{UNKNOWN}} + \mathcal{O}\left(\frac{1}{\Lambda^4}\right)$$

~ Theory is linear in Wilson coeff



What we are doing is NOT TESTING THE SM, but testing another theory that deviates from the SM.

→ The graph shows that the calculation is for EACH OBSERVEABLE and what I'll have is a linear system.

~→ In the system I know everything and I solve for C_i

Why this is so powerful?

- Imagine C_1 is non-zero ⤳ but the uncertainty is very large! I cannot say anything.

What do I do? I can combine the measurem. using the other observables
That's what in the picture. ⤳ combination.

$$\Delta O_1 = O_1^{EXP} - O_1^{SM} = \frac{1}{\Lambda^2} a_{1,1}^{(6)}(\mu)\, C_1(\mu) + \frac{1}{\Lambda^2} \cancel{a_{1,2}^{(6)}(\mu)}\, C_2(\mu) +$$

$$\Delta O_2 = O_2^{EXP} - O_2^{SM} = \frac{1}{\Lambda^2} a_{2,1}^{(6)}(\mu)\, C_1(\mu) + \frac{1}{\Lambda^2} \cancel{a_{2,2}^{(6)}(\mu)}\, C_2(\mu) +$$

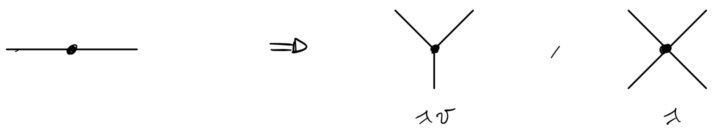
$$\Delta O_3 = O_3^{EXP} - O_3^{SM} = \frac{1}{\Lambda^2} \cancel{a_{3,1}^{(6)}(\mu)}\, C_1(\mu) + \frac{1}{\Lambda^2} a_{3,2}^{(6)}(\mu)\, C_2(\mu) +$$

$$\vdots$$

Power: Relate measurements done in different experiments and putting them togheter in one frame

⤳ Which theory has C_1 with $a_{1,2}^{(6)}(\mu)$, $a_{2,1}^{(6)}(\mu)$ and C_2 ...
So you search for th corresponding UV theory

SM Locking



Predicting power of the SM? ONCE YOU KNOW THE MASSES OF THE PARTICLES YOU KNOW EVERYTHING

- If you know Fermion mass → You know Yukawa
- If you know W, Z masses → You know g, g'
  (v you know from μ decay)

- If you know h mass → You know λ

You locked the interactions (3-points, 4-points) once you know the 2 point

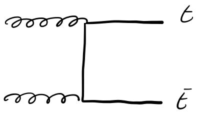
EFT is unloking the SM → Yukawa, mass

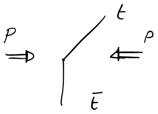
To motivate the build of the FCC the EFTs are used.

Assumptions are made : the CKM structure is maintained at dim-6 and we don't impose more symmetries.

Then we need to identify observables to measure at colliders.

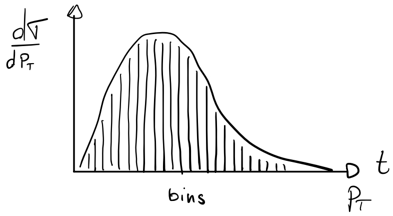
$t\bar{t}$





We can measure the differential σ

Like



With measurements we are getting an histogram
$\rightsquigarrow$ Each bin is an observable cause has its own measured value and uncertainty

Now we consider the INSERTION of a new operator : CHROMOMAGNETIC OPERATOR

$$\bar{Q}\tilde{\phi}\,T^a\sigma^{\mu\nu}t_R\,G^a_{\mu\nu}$$

- it looks like Yukawa $(\bar{Q}\tilde{\phi}t_R)$ but there is a dipole operator $\sigma^{\mu\nu}$ and Gluon Field strenght

The operator is dim6

- Q – 3/2
- $\tilde{\phi}$ – 1
- t_R – 3/2
- $G_{\mu\nu}$ – 2

when the higgs gets its vev $\phi = \begin{pmatrix} 0 \\ v+h \end{pmatrix}$

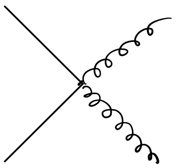
developing also $G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + ig f^{abc} A^b_\mu A^c_\nu$

NON ABELIAN

So i'll end up with a term like

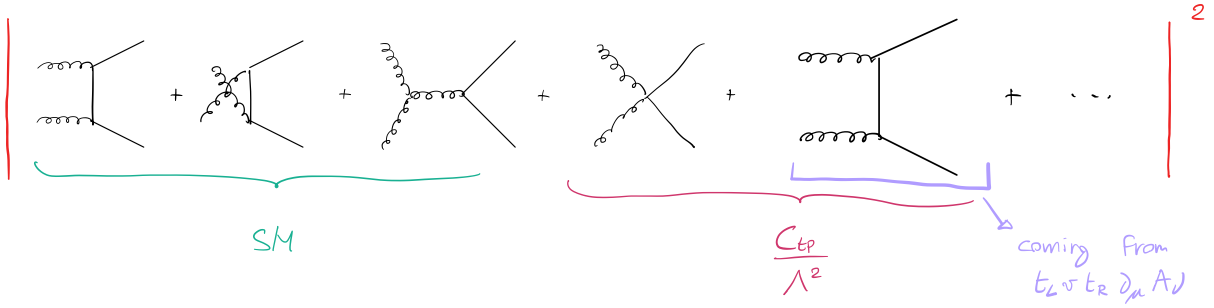
$$\overset{3/2}{v}\,\overset{3/2}{t}\,\overset{1}{t}\,\overset{1}{A}\,A$$

$\hookrightarrow$ dim 5



NOT IN THE SM

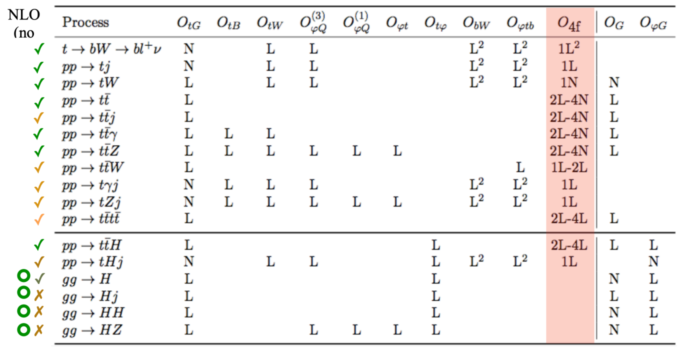
So now I have a QCD diagram to add at the previous



Squaring the new term only $\left(\to \frac{1}{\Lambda^4} \right)$ we go beyond the accuracy of an EFT expansion.

Our accuracy is still in $\frac{1}{\Lambda^2}$, that is a dim 6 term. If adding the higher term we get something non consistent $\to$ We use it only for check.

So what I do is the $Ob^{TH} - Ob^{EXP}$ that we discussed



NLO (no	Process	O_{tG}	O_{tB}	O_{tW}	$O_{\varphi Q}^{(3)}$	$O_{\varphi Q}^{(1)}$	$O_{\varphi t}$	$O_{t\varphi}$	O_{bW}	$O_{\varphi tb}$	O_{4f}	O_G	$O_{\varphi G}$
✓	$t \to bW \to bl^+\nu$	N		L	L				L^2	L^2	1L^2		
✓	$pp \to tj$	N		L	L				L^2	L^2	1L		
✓	$pp \to tW$	L		L	L				L^2	L^2	1N	N	
✓	$pp \to t\bar{t}$	L									2L-4N	L	
✓	$pp \to t\bar{t}j$	L									2L-4N	L	
✓	$pp \to t\bar{t}\gamma$	L	L	L							2L-4N	L	
✓	$pp \to t\bar{t}Z$	L	L	L	L	L	L				2L-4N	L	
✓	$pp \to t\bar{t}W$	L								L	1L-2L		
✓	$pp \to t\gamma j$	N	L	L	L				L^2	L^2	1L		
✓	$pp \to tZj$	N	L	L	L	L	L		L^2	L^2	1L		
✓	$pp \to t\bar{t}t\bar{t}$	L									2L-4L	L	
✓	$pp \to t\bar{t}H$	L						L			2L-4L	L	L
✓	$pp \to tHj$	N		L	L			L	L^2	L^2	1L		N
○ ✓	$gg \to H$	L						L				N	L
○ ✗	$gg \to Hj$	L						L				L	L
○ ✗	$gg \to HH$	L						L				N	L
○ ✗	$gg \to HZ$	L			L	L	L	L				N	L

L: leading order
N: Next lead ord

Chromodym OPERATOR
enters many observable

- If an entrance is activated means that the corresponding a_i is NON ZERO

What is the aim of this? What is the minimal requirement of this table to satisfy?

$\to$ I have to constrain a COEFFICIENT per each column, so at least has to happend

If one column is empty I have nothing to fix the operator with.
I need to have at LEAST one entry per each column, the more entries the better it is.

Now let's do this game with 3 operators :

$$O_{t\phi} = y_t^3 (\phi^+\phi)(\bar{Q}t)\tilde{\phi}$$
$$O_{\phi G} = y_t^2 (\phi^+\phi) G^A_{\mu\nu} G^{A\mu\nu}$$
$$O_{tG} = y_t g_s (\bar{Q}\sigma^{\mu\nu}T^A t)\tilde{\phi} G^A_{\mu\nu}$$

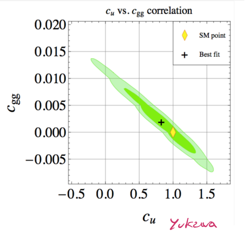
and I take 4 observable

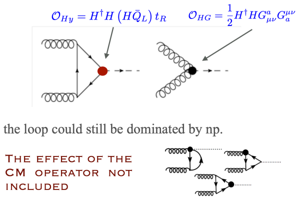
$gg \to t t H$ $\quad$ $gg \to H + j$

$gg \to H$ $\quad$ $gg \to H H$

then I do a global Fit

What I find is





the loop could still be dominated by np.

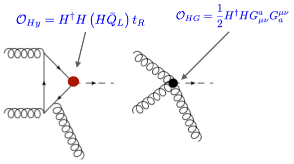
THE EFFECT OF THE CM OPERATOR NOT INCLUDED

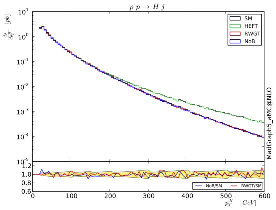
Here I have $gg \to H$ so 1 measurement and 2 observables.

$\sim\!\to$ FLAT DIRECTION $\sim$ I constrain the SUM but not the DIFFERENCE.

Now I add observables : Higgs production with an extra jet

$$gg \to H + j$$





Now somenthig is happening: the 2 diagrams behave differently at high P_T.

The wavelenght is small.

So it's small enough for the jet to "see" the loop.

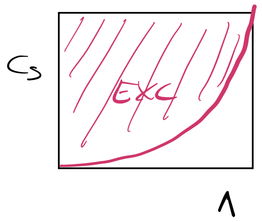
$\Rightarrow$ Resolve Flat directio

And I continue adding info doing a global Fit.

Running Wilson coefficients

Suppose we measure $\gamma\gamma$ production at our colliders

→ Use invariant mass to set limit on C_S $\qquad \frac{C_S}{\Lambda^2} < 1\,\text{TeV}^{-2}$



In order for the limit to be valid we have to exclude that region.

Why excluded because $\Delta Ob = Ob^{Th} - Ob^{Ex} = \frac{C_S}{\Lambda^2}\, O \cdot a.$

Since $\Delta Ob = 1\,\text{TeV}^{-1}$

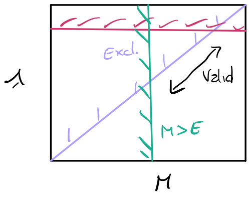
So $\frac{C_S}{\Lambda^2} < 1\,\text{TeV}^{-1}$

→ This means that if the new scale is near the SM then the C_S coefficients are more excludeable

If the scale of the new physics goes higher, then we cannot exclude less C_S because they can be as high as they want since Λ is big

Take ϕ model and we match $\qquad \frac{C_S}{\Lambda^2} = \frac{\lambda^2}{M^2} \qquad C_T = 0$

Then the limit goes to λ^2/M^2 but the dependence is now linear



⊙ Beyond some value λ is non perturbative

→ Theory is not predictive, can't match

⊙ We measure at some energy/mass bin, E : must be less then M