

Flavour Physics in the *quark sector*



Phenomenology and Experimental Flavor Physics

2 models

1° Module: weak interaction in the quark sector and CP violation. ~ Angelo Carbone

2° Module: Flavor physics in the leptonic sector ~ Maximiliano Sisti

Books: ① "CP violation" Bigi and Sanda

② "Modern Particle Physics", Mark Thomson

Exam: "some questions"
"Best of the 2" → try 2 times
"Oral Exam"

Objectives:

- history of Flavor physics
- Measurements from colliders and neutrino experiments

Meaning: Flavor Physics is the description of effects related to the

What we will do:

- Quantum Mechanics oscillation in the K, D and B meson systems and exper. results
- CP violation in the K, D, B mesons decay.
- Unitarity Triangle and CKM matrix
- Rare K, D, B decays and experimental results.
- Indirect search for New Physics with FP

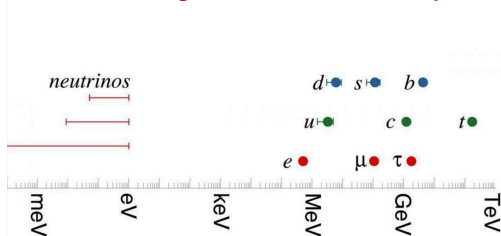
Introduction - HADRONIC WEEK INTERACTION

Some fundamental questions in flavour physics

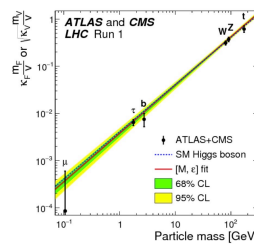
- Why three generations?
- What's the origin of the mass hierarchy?
- What's the origin of the coupling structure?
- What's the origin of the baryon asymmetry of the universe?
- What's dark and energy matter made of?

Discrepancies from the Standard Model could reveal the way to find the right answers

What's the origin of the mass hierarchy?

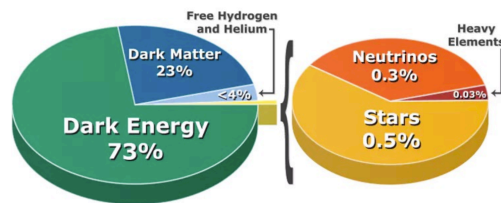


What's the origin of the coupling structure?



Higgs boson couplings to the different particles as a function of particle mass

What's dark and energy matter made of



What is flavour physics?

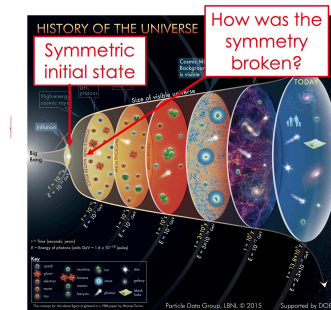
The Flavour, in particle physics, is a species of elementary particle. It is a quantum number of elementary particle. In quantum chromodynamics, flavour is a global symmetry.

In the electroweak theory the symmetry is broken and a flavour changing process exists

- decay of quarks
- neutrinos oscillations

“The term flavour was first used in particle physics in the context of the quark model of hadrons. It was coined in 1971 by Murray Gell-Mann and his student at the time, Harald Fritzsch, at a Baskin- Robbins ice-cream store in Pasadena. Just as ice cream has both colour and flavour so do quarks. ”

Nowadays we know that there's no evidence of primary antimatter on the scale of the observable universe. What led to the disappearance of antimatter assuming an initial symmetric state? How big the asymmetry should have been?



Mainstream explanation

Antimatter and matter particles annihilated massively in the early universe, but a tiny fraction of matter was left over: every 10 billion particles, a handful was not annihilated away. The radiation produced by this gigantic initial annihilation is what we see today as the big bang afterglow: the cosmic microwave background.

Sakharov conditions

In 1967 Sakharov enumerated the three conditions which are necessary for the dynamical evolution of an initially symmetric to a matter-dominated universe

1. Baryon number should not be conserved

- Otherwise there's no way to produce an excess of baryons

2. Charge (C) and Charge-Parity (CP) should not be conserved

- Interactions which produce more baryons should not be counterbalanced by interactions which produce more anti-baryons

3. Interactions must be out of thermal equilibrium

- Otherwise the baryonic asymmetry is diluted by inverse processes

In a few words: the universe is asymmetric because the baryon number is not conserved in C- and CP- violating processes giving rise to more baryons than antibaryons in the expanding universe.

Can we explain the asymmetry by known Physics?

Qualitatively: yes

– The Standard model in principle contains all the necessary ingredients

It is possible to derive the ratio of the number of baryons to that of photons in the universe

$$\eta = \frac{n_B}{n_\gamma} \sim \frac{(m_t^2 - m_u^2)(m_t^2 - m_c^2)(m_c^2 - m_u^2)(m_b^2 - m_s^2)(m_b^2 - m_d^2)(m_s^2 - m_d^2) J}{M^{12}}$$

Where J is almost 3×10^{-5} is the **Jarlskog** invariant quantifying the size of CP violation in the Standard Model and $M \approx 100$ GeV is the electroweak scale at which the baryon asymmetry freezes out

Quantitatively: no

The previous equation gives $\eta \approx 10^{-19}$, whereas using Planck experimental data on cosmic microwave background one gets

$$\eta = (6.04 \pm 0.08) \times 10^{-10}$$

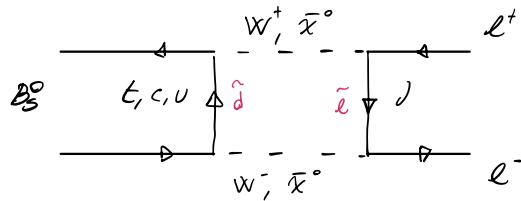
This is off by 10 orders of magnitude!

CP violation in the Standard Model is too small— **strong indication that new sources of CP violation should exist in some beyond-the-SM physics.**

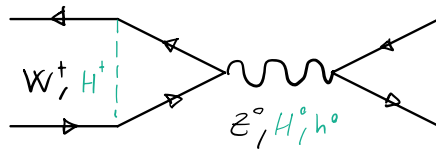
Indirect Search of New Physics.

Observed deviation From values expected according to the Standard Model will indirectly hint to the existence of New Physics. Why?

Rare Decay



you can describe the same diagram with new particles



you

When you describe the probability For example of $B_s^0 \rightarrow \mu^+ \mu^-$ you have to square the sum of SM and BSM diagram contributions.

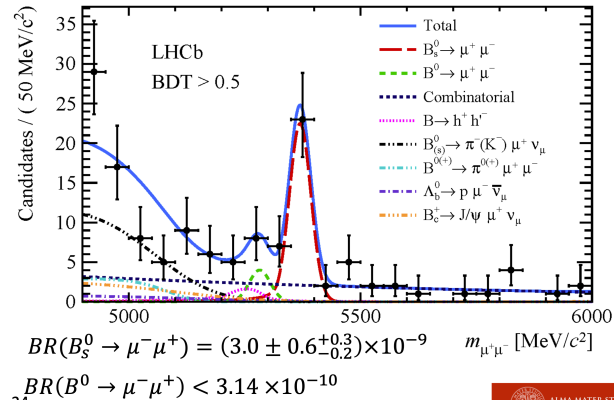
SM Predict This

$$\frac{d\sigma^{B_s^0 \rightarrow \mu^+ \mu^-}}{d\Omega} = \left| \left(\begin{array}{c} \text{SM diagrams} \\ + \text{BSM diagrams} \end{array} \right) \right|^2$$

The SM diagrams (top) include box diagrams with W bosons and Z bosons. The BSM diagrams (bottom) include diagrams with new particles like χ and A^0, H^0 .

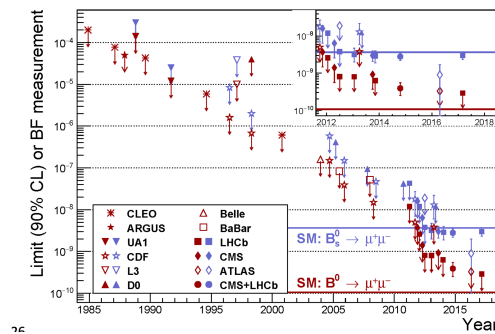
You measure significantly different From SM

Let's see the plot for the invariant mass of $B \rightarrow \mu \mu$ at LHCb



But SM works perfectly.

Limit at 90% CL or BF measurements
(IF you don't measure you put a constrain)



Blue line is the SM prediction $\rightarrow$ No evidence that there something new

Where practice FP?

- LHCb with high precision
 - complemented with ATLAS/CMS
- Belle-II
 - e^+e^- experiment
- LHCb upgrade

Lagrangian of the Standard Model

The complete $\mathcal{L}$

$$\mathcal{L} = \mathcal{L}_K + \mathcal{L}_H + \mathcal{L}_Y$$

$$\mathcal{L}_H = \underbrace{-m_e e \bar{e}}_{\text{Mass Term}} - \underbrace{\frac{m_e}{\sqrt{2}} e \bar{e} h}_{\text{coupling with Higgs}}$$

$$\mathcal{L}_D + \dots + \dots + \mathcal{L}_{K(e^-)} + \dots$$

$$\mathcal{L}_D = -\frac{g}{\sqrt{2}} \left[\underbrace{\bar{\nu}_{e,L}}_{\text{INTERACTION BASES}} \gamma^\mu \underbrace{W_{\mu}^+}_{\text{INTERACTION BASES}} e_L + \bar{e}_L \gamma^\mu W_{\mu}^- \nu_{e,L} \right]$$

The Interaction bases are the same as the mass bases $\leadsto$ Just in the ELECTRON SECTOR

$\Rightarrow$ LEPTON UNIVERSALITY [universal coupling strength in the weak interaction]

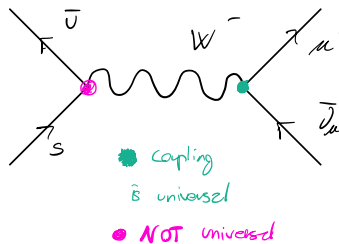
This is seen in the measurements of the

$$\begin{aligned} BR(\tau^0 \rightarrow e^+ e^-) &= 3.6\% \\ BR(\tau^0 \rightarrow \mu^+ \mu^-) &= 3.6\% \\ BR(\tau^0 \rightarrow e^+ e^-) &= 3.6\% \end{aligned}$$

Here we study something that conceptually is totally different, the interactions of QUARKS

Cabibbo Angle

$$K^-(\bar{u}s) \rightarrow \mu^- \bar{\nu}_\mu$$



The coupling is

$$\frac{g_W}{\sqrt{2}} \sin \theta_c \sim 0.22 \cdot \frac{g_W}{\sqrt{2}}$$

There is a suppression. Indeed if we measure The $BR(\pi^- \rightarrow \mu^- \bar{\nu}_\mu)$ we measure something else with the coupling in the $\pi^- \rightarrow \mu^- \bar{\nu}_\mu$ being

$$\frac{g_W}{\sqrt{2}} \cos \theta_c \sim 1 \cdot \frac{g_W}{\sqrt{2}}$$

Cabibbo explained the NOT universality $\Rightarrow$ The Lagrangian is NOT in the MASS BASIS!

Let's write the Lagrangian

$$\mathcal{L}_{K(q)} = -\frac{g_w}{\sqrt{2}} \left[\bar{u}_L^i \gamma^\mu \underbrace{V_{ij}}_{\substack{\text{CKM} \\ \text{UP-TYPE quark} \\ i \in \{u, c, t\}}} \underbrace{d_L^j}_{\substack{\text{DOWN TYPE quark} \\ j \in \{d, s, b\}}} W_\mu^+ + \underbrace{\bar{d}_L^i}_{\substack{\text{For now just like that}}} \gamma^\mu \underbrace{V_{ij}^*}_{\substack{\text{CKM} \\ \text{DOWN TYPE quark} \\ j \in \{d, s, b\}}} u_L^j W_\mu^- \right]$$

INTERACTION BASIS

Let's write the Higgs part for the quarks

$$\mathcal{L}_{H(q)} = -m_u^i \bar{u}^i u - m_d^j \bar{d}^j d - \frac{m_u^i}{\sqrt{2}} \bar{u}^i u h - \frac{m_d^j}{\sqrt{2}} \bar{d}^j d h$$

$\leadsto$ In here we used the mass basis for the quarks

We can define the INTERACTION BASIS for the quarks

$$\tilde{d}^i = \sum_j V_{ij} d_L^j$$

Now taking the $\mathcal{L}$ for quark u only

$$\mathcal{L}_{K(u)} = -\frac{g_w}{\sqrt{2}} \left[\bar{u} \gamma^\mu \left(V_{ud} d_L + V_{us} s_L + V_{ub} b_L \right) W_\mu^+ + \text{h.c.} \right]$$

FOR THE UP TYPE QUARK (Note the index i)

$$\mathcal{L}_{K(u)} = -\frac{g_w}{\sqrt{2}} \left[\bar{u}_L^i \gamma^\mu \underbrace{\tilde{d}_L^i}_{\substack{\text{INTERACTION BASIS}}} W_\mu^+ + \text{h.c.} \right]$$

$\leadsto$ So we have learned that the interaction basis are NOT the real quarks, but a combination of them multiplied by their probability

P.S. In the diagrams we use the mass basis

Note: Note that the UP-TYPE quark is not rotated. $\leadsto$ chosen to be fixed.

The CKM Matrix

We define the transformation between the Interaction Basis and Mass Basis.

→ In this definition we decide that UP-TYPE quarks are the only that remain the same.

$$\begin{pmatrix} \hat{d} \\ \hat{s} \\ \hat{b} \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

SPINOR OF INTERACTION BASIS MASS BASIS

Properties: $V^\dagger V = \mathbb{1}$ († being transp-conjugate)

3x3 Matrix but just 4 PARAMETERS)

Parametrization:

$$V_{CKM} = \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

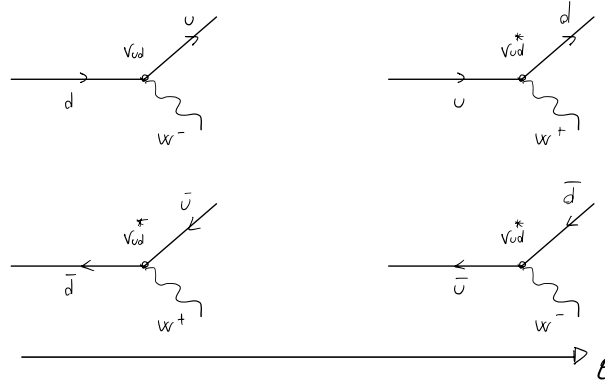
with $\lambda = 0.2260 \pm 0.0001$ ($\sin \theta_c$)

$A = 0.826 \pm 0.012$

$\rho = 0.152 \pm 0.014$

$\eta = 0.357 \pm 0.010$

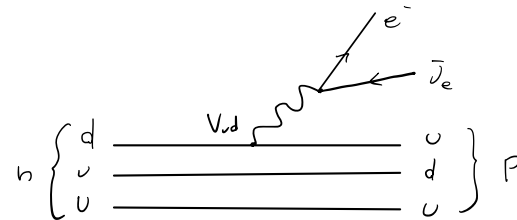
Pictorial Explanation of the CKM Matrix Element



Here a small overview of how to measure various element of V_{CKM}

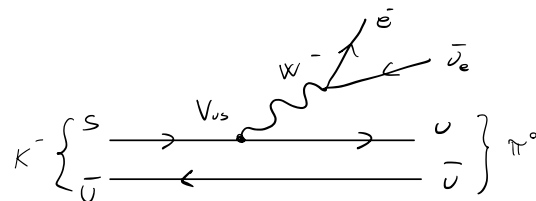
- $|V_{ud}| = (0.97431 \pm 0.00012)$

Decay used: β -decay



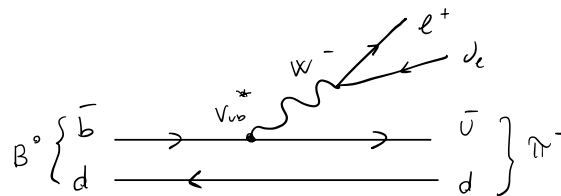
- $|V_{us}| = (0.22514 \pm 0.0055)$

Decay used: $K^- \rightarrow \pi^0 e^- \bar{\nu}_e$



- $|V_{ub}| = (0.00365 \pm 0.00010)$

Decay used: $B^0 \rightarrow \pi^- e^+ \nu_e$



$$V_{ub} = |V_{ub}| e^{-i\gamma}$$

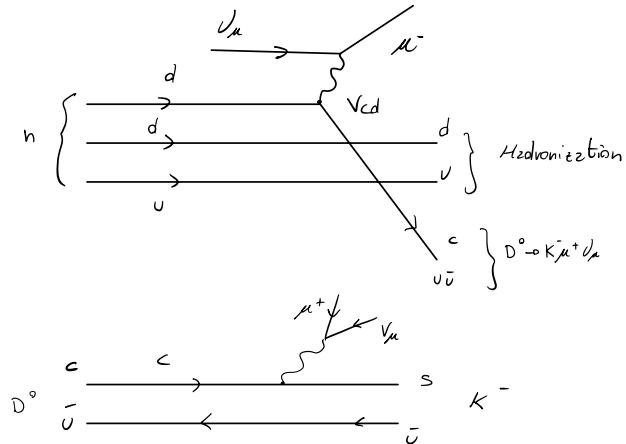
$$V_{ub} = A \lambda^3 (\rho - i\eta) \quad \gamma = \arg\left(\frac{\eta}{\rho}\right) = (66.8 \pm 2.0)^\circ$$

$$\Rightarrow V_{ub} = (0.00365 \pm 0.00010) e^{-i(66.8 \pm 2.0)^\circ}$$

- $|V_{cd}| = (0.218 \pm 0.004)$

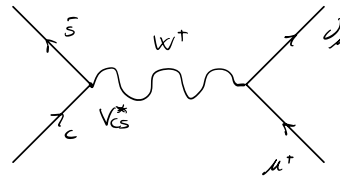
Decay used: $D \rightarrow \pi \ell \nu$

$D \rightarrow K \ell \nu$



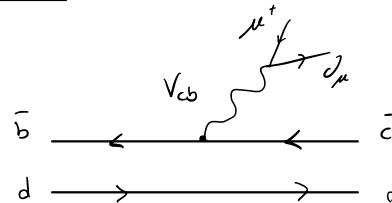
- $|V_{cs}| = (0.97344 \pm 0.00012)$

Decay used: $D_s^+ \rightarrow \mu^+ \nu_\mu$

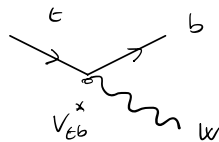


- $|V_{cb}| = (0.04271 \pm 0.00065)$

Decay used: $B^0 \rightarrow D^+ \mu^- \nu_\mu$



- $|V_{tb}| = (0.999112 \pm 0.000024)$



Just from top quarks because it decays before hadronization

- $|V_{td}| = (0.00869 \pm 0.00017)$

Using TOP physics

$\Rightarrow V_{td}$ is complex $\Rightarrow$ Measured with $B^0 \leftrightarrow \bar{B}^0$ oscillation

- $|V_{ts}| = (0.04124 \pm 0.00056) \Rightarrow$ Measured with $B_s^0 \leftrightarrow \bar{B}_s^0$ oscillations

Unitarity Triangle

$$V^\dagger V = \mathbb{1}$$

$$\begin{pmatrix} V_{ud}^* & V_{cd}^* & V_{td}^* \\ V_{us}^* & V_{cs}^* & V_{ts}^* \\ V_{ub}^* & V_{cb}^* & V_{tb}^* \end{pmatrix} \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\Rightarrow V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0$$

In order to build the triangle we can divide by $V_{cb}^* V_{cd}$

$$\frac{V_{ub}^* V_{ud}}{V_{cb}^* V_{cd}} + 1 + \frac{V_{tb}^* V_{td}}{V_{cb}^* V_{cd}} = 0$$

only 2 numbers are imaginary

Expliciting V_{ub} :

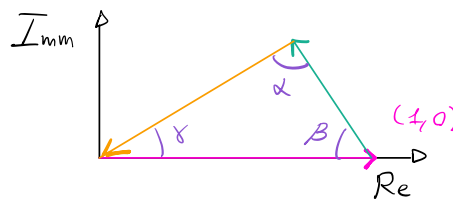
$$V_{ub} = |V_{ub}| \cdot e^{-i\gamma}$$

$$V_{td} = |V_{td}| \cdot e^{-i\beta}$$

Plugging in

$$\frac{1}{\underbrace{V_{cb}^* V_{cd}}} - \frac{|V_{ub}| |V_{ud}|}{|V_{cb}^*| |V_{cd}|} e^{i\gamma} - \frac{|V_{tb}^*| |V_{td}|}{|V_{cb}^*| |V_{cd}|} e^{-i\beta} = 0$$

Graphical Representation



$$\gamma \equiv \text{Arg} \left[\frac{-V_{ub}^* \cdot V_{ub}}{V_{cb}^* \cdot V_{cd}} \right]; \quad \beta \equiv \text{Arg} \left[\frac{-V_{cb}^* \cdot V_{cd}}{V_{tb}^* \cdot V_{td}} \right]; \quad \alpha = \text{Arg} \left[\frac{-V_{td} \cdot V_{tb}^*}{V_{ud} \cdot V_{ub}^*} \right]$$

($z = |z| e^{i \text{Arg}(z)}$)

Lets analyze one of the side of the triangle

$$\frac{V_{ub}^* \cdot V_{ub}}{V_{cb}^* \cdot V_{cd}} = \frac{A \lambda^3 (\bar{f} + i\bar{q}) (1 - \lambda^2/2)}{A \lambda^2 (-\lambda)} = - \left[\bar{f} \left(1 - \frac{\lambda^2}{2} \right) + i\bar{q} \left(1 - \frac{\lambda^2}{2} \right) \right] = -[\bar{f} + i\bar{q}]$$

Defining $\bar{f} = f \left(1 - \frac{\alpha^2}{2}\right)$, $\bar{\eta} = \eta \left(1 - \frac{\alpha^2}{2}\right)$

EXERCISE 1

Compute $\frac{|V_{cb}^*| |V_{cd}|}{|V_{cb}| |V_{cd}|}$

IMPORTANT NOTE:

We know that CP is violated $\leadsto$ Result in the CKM matrix? Should happen that the CKM matrix is COMPLEX
Why?

$\Rightarrow$ Look at CP VIOLATION IN THE LAGRANGIAN

Let's take a term, the kinematic part for the quark

$$\mathcal{L}_{(q)} = \frac{-g_w}{\sqrt{2}} \left[\bar{u}_L \gamma^\mu (V_{ud} d_L + V_{us} s_L + V_{ub} b_L) W_\mu^+ + (V_{ud}^* \bar{d}_L + V_{us}^* \bar{s}_L + V_{ub}^* \bar{b}_L) \gamma^\mu u_L W_\mu^- \right] + \text{h.c.}$$

If we apply the CP operator (change particle with antiparticle)

$$\mathcal{L}_{(q)} = \frac{-g_w}{\sqrt{2}} \left[(V_{ud} \bar{d}_L + V_{us} \bar{s}_L + V_{ub} \bar{b}_L) \gamma^\mu u_L W_\mu^- + \bar{u}_L \gamma^\mu (V_{ud}^* d_L + V_{us}^* s_L + V_{ub}^* b_L) W_\mu^+ \right] + \text{h.c.}$$

If you want that CP is conserved, looking at the GREEN ELEMENTS, we need to have $V_{ud} = V_{ud}^*$ $\leadsto V_{ud} \in \mathbb{R}$

But we OBSERVE CP violation $\leadsto V_{ud} \neq V_{ud}^* \leadsto V_{CKM} \in \mathbb{C}$

Let's introduce $J \equiv$ JARLKO INARIANT

$$J = \frac{1}{2} (V_{22} V_{33} V_{23}^* V_{32}^*) = \dots = A^2 \lambda^6 \eta = 2 \cdot \text{Area Unitarity Triangle}$$

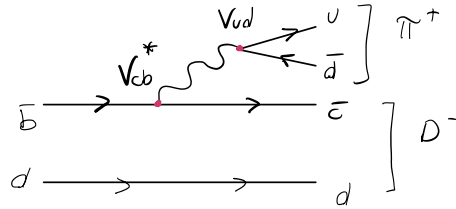
$$\approx 3 \cdot 10^{-5}$$

$\leadsto$ If $J \neq 0 \Rightarrow$ CP is violated

Examples

Example 1

$$B^0 \rightarrow D^- \pi^+$$



$$BR(B^0 \rightarrow D^- \pi^+) \propto p^* |V_{cb}^*|^2 |V_{ud}|^2$$

$$|V_{cb}^*|^2 |V_{ud}|^2 = \left(1 - \frac{1}{2}\right) (A\lambda^2)^2$$

$$\text{Decay Width } \Gamma_{B^0 \rightarrow D^- \pi^+} \propto p^* \propto \frac{1}{2m_{B^0}} \left\{ [m_{B^0}^2 - (m_{\pi^+} + m_{D^-})^2] [m_{B^0}^2 - (m_{\pi^+} - m_{D^-})^2] \right\}^{1/2}$$

$$m_{B^0} = 5.279 \text{ GeV}/c^2$$

don't pretend to calculate $\rightarrow$ it's example

$$m_{D^-} = 1.870 \text{ GeV}/c^2$$

$$m_{\pi^+} = 0.140 \text{ GeV}/c^2$$

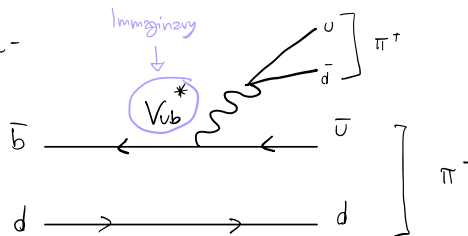
$$|V_{cb}^*|^2 |V_{ud}|^2 \sim 1.6 \cdot 10^{-3}$$

$$BR(B^0 \rightarrow D^- \pi^+) \propto P_{D\pi}^* |V_{cb}^*|^2 |V_{ud}|^2 \sim 2.7 \cdot 10^{-3}$$

$$PDG \quad 2.5 \cdot 10^{-3}$$

Example 2

$$B^0 \rightarrow \pi^+ \pi^-$$

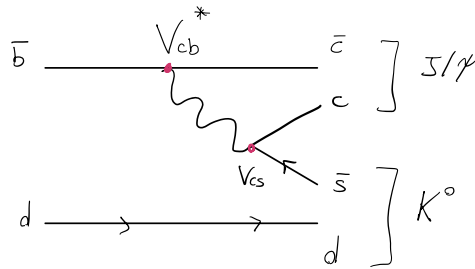


$$BR(B^0 \rightarrow \pi^+ \pi^-) \propto P_{\pi^+ \pi^-}^* |V_{ub}^*|^2 |V_{ud}|^2 \sim 2.4 \cdot 10^{-5}$$

$$|V_{ub}^*|^2 |V_{ud}|^2 = \left(1 - \frac{1}{2}\right)^2 \cdot (A\lambda^3)^2 (1 + \lambda^2) \sim 1.16 \cdot 10^{-5}$$

Example 3

$$B^0 \rightarrow \bar{S}/\gamma K^0$$



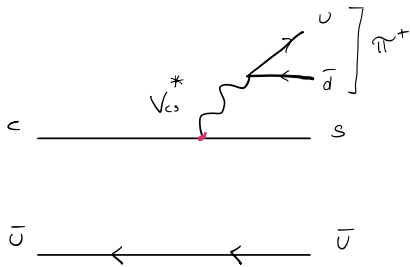
$$|V_{cs}|^2 |V_{cb}^*|^2 = \left(1 - \frac{1}{2}\right)^2 (A \lambda^2)^2$$

$$= 1.6 \cdot 10^{-3}$$

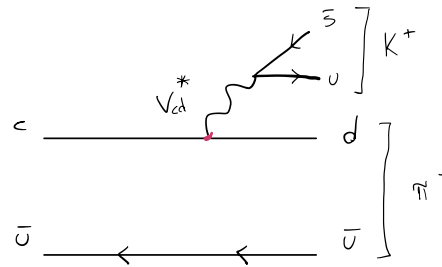
$$\text{BR}(B^0 \rightarrow \bar{S}/\gamma K^0) \propto P^* |V_{cs}|^2 |V_{cb}|^2 \sim 1.0 \cdot 10^{-3}$$

Example 4

$$D^0 \rightarrow K^- \pi^+$$



$$D^0 \rightarrow K^+ \pi^-$$



$$R = \frac{\text{BR}(D^0 \rightarrow K^- \pi^+)}{\text{BR}(D^0 \rightarrow K^+ \pi^-)} = \frac{\left(1 - \frac{1}{2}\right)^2 \left(1 - \frac{1}{2}\right)^2}{\lambda^2 \lambda^2} \sim \frac{1}{\lambda^4} \sim \frac{1}{3 \cdot 10^{-3}}$$

The Neutral Kaon System $K^0(d\bar{s})$, $\bar{K}^0(\bar{d}s)$

The first experimental observation of CP violation was made in the neutral kaon sys. These are produced copiously in strong interactions:

- $\pi^- + p \rightarrow \Lambda + K^0$
- $p + \bar{p} \rightarrow K^+ + \bar{K}^0 + \pi^-$

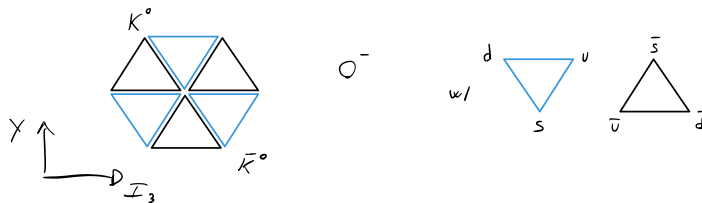
We have:

K^0 neutral meson $|d\bar{s}\rangle$
 $m(K^0) = 497.6 \text{ MeV}/c^2$

The antiparticle $\bar{K}^0 |\bar{d}s\rangle$
 $m(\bar{K}^0) = 497.6 \text{ MeV}/c^2$

$\Rightarrow$ Lightest hadrons containing strange quark

Scheme:



$\leadsto K^0$ decays in leptons and hadrons only via weak interactions (are lightest and since are light, kinematically are allowed $e/\mu/\pi$ mesons with s quark)

Since K^0 is neutral $\leadsto$ Oscillate into his antiparticle $\Rightarrow$ Propagate as a linear comb of K^0 and $\bar{K}^0$.

We have Flavour Eigenstates $K^0(d\bar{s})$, $\bar{K}^0(\bar{d}s)$ $\leadsto$ Interact with weak interactions

BECAUSE OF OSCILLATION $\Rightarrow$ Have a Mass eigenstate K_S, K_L $\Rightarrow$ Measurable

Peculiarity of kaon system is the lifetime

$$\begin{aligned} \bullet \tau(K_S) &\sim 0.9 \cdot 10^{-10} \text{ s} \\ \bullet \tau(K_L) &\sim 0.5 \cdot 10^{-7} \text{ s} \end{aligned}$$

This is the only system where the lifetime is so different!

The lifetime of B_s, D^0 for example have comparable lifetimes.

STATIC CASE

$$K^0, \bar{K}^0 \quad S^P = 0^-$$

~ The parity P operator

$$P|K^0\rangle = -|K^0\rangle$$

↙
Boson
State

$$P|\bar{K}^0\rangle = -|\bar{K}^0\rangle$$

~ The C operator charge the wave function onto the w.f. of the antiparticle

$$C|K^0(d\bar{s})\rangle = C|d\bar{s}\rangle = -|\bar{d}s\rangle = -|\bar{K}^0\rangle$$

↙ Convention

$$C|K^0\rangle = -|\bar{K}^0\rangle$$

$$C|\bar{K}^0\rangle = -|K^0\rangle$$

Combining them you get

$$CP|\bar{K}^0\rangle = +|K^0\rangle \quad (\text{double minus})$$

When CP is conserved you can measure the CP in the decays of the particles and infer on the decaying particle (since it's conserved)

Let's now write K_1, K_2 as a linear combination of $K^0, \bar{K}^0$

$$|K_1\rangle = \frac{1}{\sqrt{2}} (|\bar{K}^0\rangle + |K^0\rangle)$$

$$|K_2\rangle = \frac{1}{\sqrt{2}} (|K^0\rangle - |\bar{K}^0\rangle)$$

So

$$CP|K_1\rangle = \frac{1}{\sqrt{2}} (|\bar{K}^0\rangle + |K^0\rangle) = +|K_1\rangle$$

Positive Eigenstate
⇒ EVEN EIGENSTATE

$$CP|K_2\rangle = \frac{1}{\sqrt{2}} (|\bar{K}^0\rangle - |K^0\rangle) = -|K_2\rangle$$

Negative Eigenstate
⇒ ODD EIGENSTATE

Let's discuss about the decays of those particles.

DECAYS OF EIGENSTATES

The K_S do not have so much room to decay, just π are possible.

→ Pions $J^P = 0^-$

In general Kaons can decay in 2 or 3 pions (not 4, because not enough center of mass)

$$\begin{aligned} K^0 &\rightarrow \pi^+ \pi^- \\ K^0 &\rightarrow \pi^0 \pi^0 \end{aligned}$$

$$\begin{aligned} K^0 &\rightarrow \pi^+ \pi^- \pi^0 \\ K^0 &\rightarrow \pi^0 \pi^0 \pi^0 \end{aligned}$$

2 PION SYSTEM

Let's consider the system of the 2 pions

PARITY

$$P(\pi^0 \pi^0) = (-1)^L P(\pi^0) \cdot P(\pi^0) = (-1)^0 \underline{(-1)} \underline{(-1)}$$

$\sim L$ is 0 because in a system of 2 particles is 0

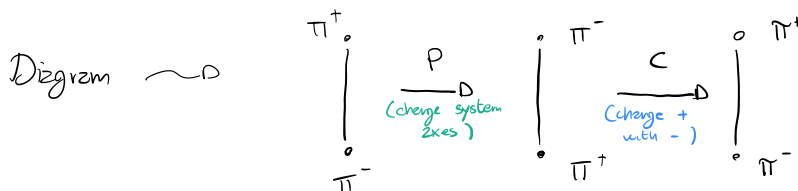
CHARGE CONJUGATION

$$C|\pi^0\rangle = \frac{1}{\sqrt{2}}(|u\bar{u}\rangle - |d\bar{d}\rangle) = |\pi^0\rangle$$

Now we apply CP operator to a pair of π^0

$$CP|\pi^0 \pi^0\rangle = |\pi^0 \pi^0\rangle$$

! For the system $|\pi^+ \pi^-\rangle$ it's different



but in the end, the same result.

$$CP|\pi^+ \pi^-\rangle = |\pi^+ \pi^-\rangle$$

3 PION SYSTEM

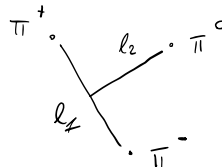
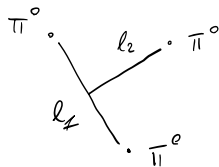
Let's consider the system of the 3 pions

$$K^0 \rightarrow \pi^+ \pi^- \pi^0, \quad K^0 \rightarrow \pi^0 \pi^0 \pi^0$$

PARITY

$$P(\pi\pi\pi) = (-1)^{l_1} (-1)^{l_2} P(\pi) \cdot P(\pi) \cdot P(\pi)$$

Why 2 angular momenta l_1, l_2 ?



Now $\vec{L} = \vec{L}_1 + \vec{L}_2$ but $\vec{S} = 0^-$ for kaon $\vec{L} = 0$
 $\leadsto$ This means that $l_1 = l_2 = l$
definition of kaon

$$\Rightarrow P(\pi\pi\pi) = (-1)^{2l} (-1)^3 = -1$$

CHARGE CONSERVATION

$$\leadsto C(\pi^+ \pi^- \pi^0) = C(\pi^+) C(\pi^-) C(\pi^0) = +1$$

$$\leadsto C(\pi^0 \pi^0 \pi^0) = C(\pi^0) C(\pi^0) C(\pi^0) = +1$$

Combining the 2

$$CP |\pi^+ \pi^- \pi^0\rangle = - |\pi^+ \pi^- \pi^0\rangle$$

$$CP |\pi^0 \pi^0 \pi^0\rangle = - |\pi^0 \pi^0 \pi^0\rangle$$

Now we also have to consider the Phase space

$$m_K - 2m_{\pi^0} \sim 220 \text{ MeV}/c^2$$

$$m_K - 3m_{\pi^0} \sim 80 \text{ MeV}/c^2$$

$\Gamma \propto P^*$ Phase Space

And $\Gamma_{\pi\pi} > \Gamma_{\pi\pi\pi}$ with $\Gamma \propto \frac{1}{c}$

Large phase space $\rightarrow$ Faster Decay

$$\Gamma_{\pi\pi\pi} > \Gamma_{\pi\pi}$$

IF CP IS CONSERVED

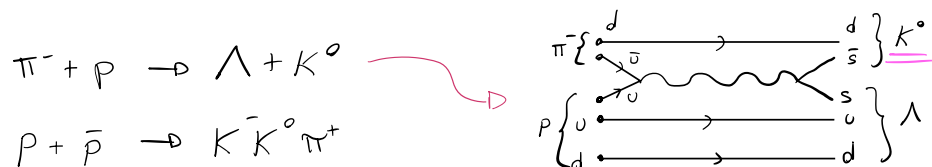
$$|K_1\rangle \rightarrow \pi\pi \quad |K_1\rangle = |K_S\rangle$$

$$|K_2\rangle \rightarrow \pi\pi\pi \quad |K_2\rangle = |K_L\rangle$$

Now let's test our expectation with an experiment.

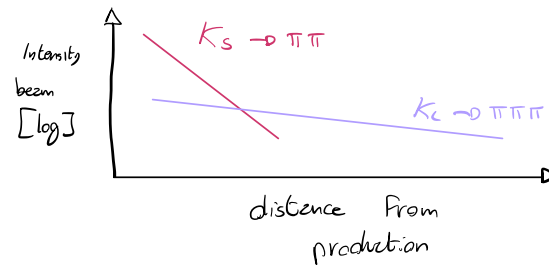
EXPERIMENT

We want to produce K^0 . To do so, we can start from a beam of proton and antiproton or a fixed target experiment with pion and fixed proton.



As said before we can write K^0 and $\bar{K}^0$ as a combination of K_S, K_L and if you let the beam travel enough distance, you'll have that, given the short decay time, all the K_S will disappear from the beam, and you will remain with pure K_L .

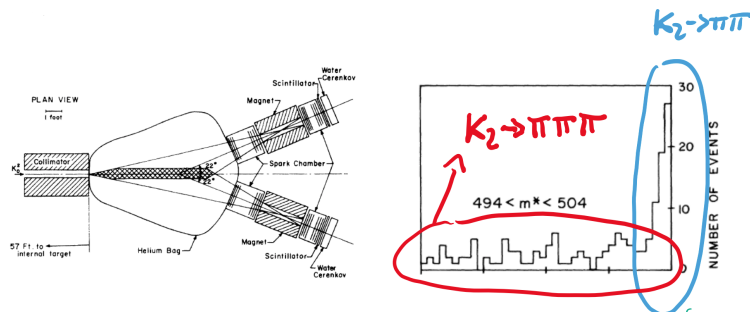
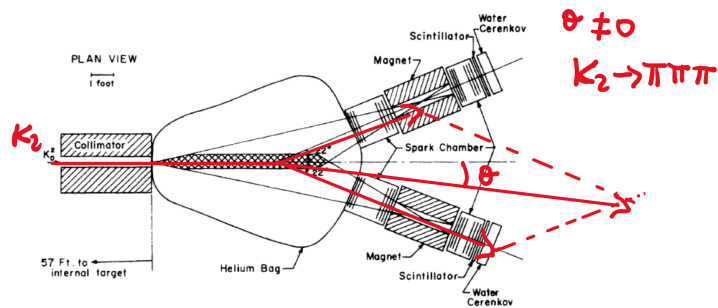
Graphically



=> We expect that, Far away from the production point we expect only 3π

THE CROMIN and FITCH EXPERIMENT

Conceptually very simple: they looked for (CP-violating) $K_2 \rightarrow \pi^- \pi^+$ decays 20 meters away from K^0 production point

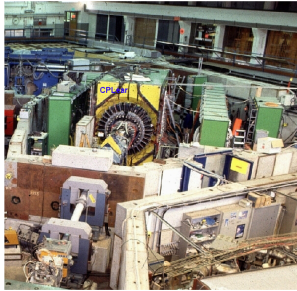


Excess of $K_2 \rightarrow \pi\pi$: impossible without CP violation

also other experiments: CPLEAR

Neutral kaon mixing: CPLEAR experiment

The CPLEAR experiment used the antiproton beam of the LEAR facility - which operated at CERN from 1982 to 1996



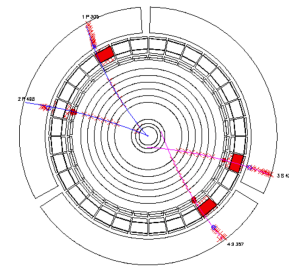
Low-Energy Antiproton Ring (LEAR)



Built in 1982 and operated until 1996

A typical $K^0 \rightarrow \pi^- \pi^+$ @CPLEAR

Run no 13470 Event no: 2455

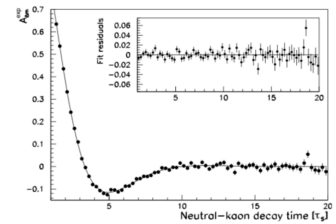
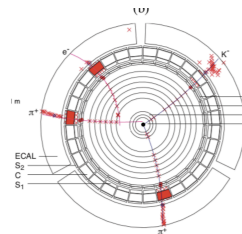


Kaon production at CPLEAR

$$p\bar{p} \rightarrow K^0 K^- \pi^+$$

$$p\bar{p} \rightarrow \bar{K}^0 K^+ \pi^-$$

The charge of the Kaon defines the flavour of the neutral Kaon at the time of the production.



$$A_{\Delta m}^{\text{theo}} = \frac{2e^{-\frac{\Gamma t}{2}} \cos(\Delta m \cdot t)}{e^{-\Gamma_s t} + e^{-\Gamma_L t}} \quad \text{Eur. Phys. J. C 22 (2001) 55-79}$$

$$\Delta m = (0.5295 \pm 0.0020_{\text{stat}} \pm 0.0003_{\text{syst}}) \times 10^{10} \text{ h/s,}$$

Now that we have seen the experimental result, let's describe the time evolution of a kaon system.

Time Evolution

Related to QM $\rightarrow$ evolution of states

A state $|\psi\rangle = |\psi(t=0)\rangle$ will evolve in $|\psi(t)\rangle = a(t)|\psi(t)\rangle$.

- we know that $|\langle\psi(t)|\psi(t)\rangle|^2 = 1$
- $a(t)$ is unitary time-evolution operator

CHARGED PARTICLES CASE

Let's focus on a charged particle

$\rightarrow$ We will compute the probability of mixing with the corresponding antiparticle

$\rightarrow$ In this case: NO MIXING

$$|P^+(t)\rangle = \underbrace{e^{-iH_P t}}_{\text{Related to life time of a particle}} \underbrace{e^{-\frac{\Gamma}{2}t}}_{\text{Time Evolution for STABLE PARTICLE with } \Gamma = \Gamma_P} |P^+\rangle$$

$\sim$ charged particle P^+

Related to life time of a particle

Time Evolution for STABLE PARTICLE with $\Gamma = \Gamma_P$

If we compute the probability given a particle to find it after a certain time

$$|\langle P^+ | P^+(t) \rangle|^2 = e^{-\Gamma t} \quad \sim \text{Introduced}$$

With this definition we can consider

$$i \frac{d}{dt} |P^+(t)\rangle = \left(H_P - \frac{i\Gamma}{2} \right) |P^+(t)\rangle \quad \sim \text{Schrödinger EQUATION}$$

NEUTRAL PARTICLE CASE

Need to consider the Probability of mixing $\gg$ Weak interaction Box allows the mixing

In case of mixing it's important to know the parameters Γ, H

We know now that we have to evolve a 2-system: particle and antiparticle

$$i \frac{d}{dt} \begin{pmatrix} |P^0(t)\rangle \\ |\bar{P}^0(t)\rangle \end{pmatrix} = \sum \begin{pmatrix} |P^0(t)\rangle \\ |\bar{P}^0(t)\rangle \end{pmatrix}$$

$\hookrightarrow 2 \times 2$ matrix

With

$$\sum = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} - \frac{i}{2} \begin{pmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{21} & \Gamma_{22} \end{pmatrix}$$
$$= \begin{pmatrix} M_{11} - \frac{i}{2} \Gamma_{11} & M_{12} - \frac{i}{2} \Gamma_{12} \\ M_{21} - \frac{i}{2} \Gamma_{21} & M_{22} - \frac{i}{2} \Gamma_{22} \end{pmatrix}$$

$$\leadsto i \frac{d}{dt} |P^0(t)\rangle = \left(M_{11} - \frac{i}{2} \Gamma_{11} \right) |P^0(t)\rangle + \left(M_{12} - \frac{i}{2} \Gamma_{12} \right) |\bar{P}^0(t)\rangle$$

$$i \frac{d}{dt} |\bar{P}^0(t)\rangle = \left(M_{22} - \frac{i}{2} \Gamma_{22} \right) |\bar{P}^0(t)\rangle + \left(M_{21} - \frac{i}{2} \Gamma_{21} \right) |P^0(t)\rangle$$

Relying on CPT theorem $\rightarrow M_{11} = M_{22}$ (This is for Flavor states)

$$\Gamma_{11} = \Gamma_{22}$$

$$M_{21} = M_{12}^* \quad \Gamma_{21} = \Gamma_{12}^*$$

So, using the information coming from CPT theorem we will have

$$\sum = \begin{pmatrix} M - \frac{i}{2} \Gamma & M_{12} - \frac{i}{2} \Gamma_{12} \\ M_{12}^* - \frac{i}{2} \Gamma_{12}^* & M - \frac{i}{2} \Gamma \end{pmatrix}$$

By diagonalizing this matrix we can ROTATE the original base and go to the base of eigenvalues.

EIGENVALUES and EIGENVECTORS

$$(\Sigma - \lambda \mathbb{I}) \mathbf{v} = 0 \quad \text{with} \quad \mathbb{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\mathbf{v}_z = \begin{pmatrix} p \\ q \end{pmatrix} \quad \mathbf{v}_z = \begin{pmatrix} p \\ -q \end{pmatrix} \quad \text{nd orthonormal}$$

$$\leadsto \begin{vmatrix} M - \frac{i}{2} \uparrow - \lambda & M_{\pm z} - \frac{i}{2} \uparrow_{\pm z} \\ M_{\pm z}^* - \frac{i}{2} \uparrow_{\pm z}^* & M - \frac{i}{2} \uparrow - \lambda \end{vmatrix} = 0$$

$$\left(M - \frac{i}{2} \uparrow - \lambda \right) \left(M - \frac{i}{2} \uparrow - \lambda \right) - \left(M_{\pm z} - \frac{i}{2} \uparrow_{\pm z} \right) \left(M_{\pm z}^* - \frac{i}{2} \uparrow_{\pm z}^* \right) = 0$$

$$\text{defining} \quad F = \sqrt{\left(M_{\pm z}^* - \frac{i}{2} \uparrow_{\pm z}^* \right) \left(M_{\pm z} - \frac{i}{2} \uparrow_{\pm z} \right)}$$

$$\text{solving} \quad \left(M - \frac{i}{2} \uparrow - \lambda \right)^2 = F^2 \quad \rightarrow \quad M - \frac{i}{2} \uparrow - \lambda = \pm F$$

hence

$$\lambda_{\pm} = \left(M - \frac{i}{2} \uparrow \right)_{\pm} (F) \quad \hookrightarrow \text{Re}\{F\} + i \text{Im}\{F\}$$

$$\bullet \lambda_L^{(-)} = m_L - \frac{i}{2} \uparrow_L = \underbrace{M - \text{Re}\{F\}}_{m_L} - \frac{i}{2} \underbrace{(\uparrow + 2 \text{Im}\{F\})}_{\uparrow_L}$$

$$\bullet \lambda_H^{(+)} = m_H - \frac{i}{2} \uparrow_H = \underbrace{M + \text{Re}\{F\}}_{m_H} - \frac{i}{2} \underbrace{(\uparrow - 2 \text{Im}\{F\})}_{\uparrow_H}$$

Now we take the difference

$$\Delta m = m_H - m_L = 2 \operatorname{Re}\{F\} > 0 \quad \Rightarrow \text{If } 0 \text{ then NO OSCILLATION}$$

so Δm it's important because it's an observable

We can also define

$$\Delta \Gamma = \Gamma_L - \Gamma_H = 2 \operatorname{Im}\{F\} \quad \text{The sign is NOT DEFINITE}$$

We can now rewrite the definition using

$$M = \frac{m_H + m_L}{2} \quad \Gamma = \frac{\Gamma_H + \Gamma_L}{2}$$

EIGENVECTORS

$$\begin{pmatrix} M - \frac{i}{2}\Gamma & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^* - \frac{i}{2}\Gamma_{12}^* & M - \frac{i}{2}\Gamma \end{pmatrix} \begin{pmatrix} p \\ q \end{pmatrix} = \lambda_{\pm} \begin{pmatrix} p \\ q \end{pmatrix}$$

$$\begin{cases} \left(M - \frac{i}{2}\Gamma\right)p + \left(M_{12} - \frac{i}{2}\Gamma_{12}\right)q = \lambda_{\pm} p \\ \left(M_{12}^* - \frac{i}{2}\Gamma_{12}^*\right)p + \left(M - \frac{i}{2}\Gamma\right)q = \lambda_{\pm} q \end{cases}$$

We can take the first equation

$$\left(M - \frac{i}{2}\Gamma\right) \frac{p}{p} - \lambda_{\pm} \frac{p}{p} = - \frac{q}{p} \left(M_{12} - \frac{i}{2}\Gamma_{12}\right)$$

$$\frac{q}{p} \left(M_{12} - \frac{i}{2}\Gamma_{12}\right) = \lambda_{\pm} - \left(M - \frac{i}{2}\Gamma\right)$$

$$\frac{q}{p} \left(M_{12} - \frac{i}{2}\Gamma_{12}\right) = \left(M - \frac{i}{2}\Gamma\right) \pm F - \left(M - \frac{i}{2}\Gamma\right)$$

$$\leadsto \frac{q}{p} = \frac{\pm F}{M_{12} - \frac{i}{2} \Gamma_{12}} = \frac{\sqrt{\left(M_{12} - \frac{i}{2} \Gamma_{12}\right) \left(M_{12}^* - \frac{i}{2} \Gamma_{12}^*\right)}}{\left(M_{12} - \frac{i}{2} \Gamma_{12}\right)^{1/2}}$$

$$\Rightarrow \frac{q}{p} = \sqrt{\frac{\left(M_{12}^* - \frac{i}{2} \Gamma_{12}^*\right)}{\left(M_{12} - \frac{i}{2} \Gamma_{12}\right)}}$$

$\leadsto$ Same expression is obtained if it is used $\begin{pmatrix} p \\ -q \end{pmatrix}$

INVERSE OF MATRIX

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$AA^{-1} = \mathbb{1}$$

$$A^{-1} = \frac{1}{\det A} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

Let's now find the inverse of the matrix Q

$$Q = \begin{pmatrix} p & p \\ -q & q \end{pmatrix}$$

$$Q^{-1} = \frac{1}{-2pq} \begin{pmatrix} -q & -p \\ -q & p \end{pmatrix}$$

$$QQ^{-1} = \mathbb{1}$$

Hence

$$\begin{aligned} Q^{-1} \Sigma Q &= \frac{1}{2pq} \begin{pmatrix} q & p \\ q & -p \end{pmatrix} \begin{pmatrix} M - \frac{i}{2} \Gamma & M_{12} - \frac{i}{2} \Gamma_{12} \\ M_{12}^* - \frac{i}{2} \Gamma_{12}^* & M - \frac{i}{2} \Gamma \end{pmatrix} \begin{pmatrix} p & p \\ q & -q \end{pmatrix} \\ &= \begin{pmatrix} \lambda_L^{(-)} & 0 \\ 0 & \lambda_H^{(+)} \end{pmatrix} = \begin{pmatrix} m_L - \frac{i}{2} \Gamma_L & 0 \\ 0 & m_H - \frac{i}{2} \Gamma_H \end{pmatrix} \end{aligned}$$

So $|P_{H,L}(t)\rangle$ obey to the exponential law

$$|P^+(t)\rangle \quad m_P, \uparrow \rightarrow |P_H(t)\rangle \quad m_H, \uparrow_H; \quad |P_L(t)\rangle \quad m_L, \uparrow_L$$

$$|P_H\rangle = p|P^0\rangle + q|\bar{P}^0\rangle$$

$$|P_L\rangle = p|P^0\rangle - q|\bar{P}^0\rangle$$

} NEW BASES WHICH OBEY TO THE EXPONENTIAL LAW

In the other basis

$$|P_H\rangle + |P_L\rangle = 2p |P^o\rangle \Rightarrow |P^o\rangle = \frac{1}{2p} (|P_H\rangle + |P_L\rangle)$$

$$|P_H\rangle - |P_L\rangle = 2q |\bar{P}^o\rangle \Rightarrow |\bar{P}^o\rangle = \frac{1}{2q} (|P_H\rangle - |P_L\rangle)$$

In the end their Time evolution

$$|P_H(t)\rangle = e^{-im_H t} \cdot e^{-\frac{\Gamma_H}{2} t} |P_H\rangle$$

$$|P_L(t)\rangle = e^{-im_L t} \cdot e^{-\frac{\Gamma_L}{2} t} |P_L\rangle$$

$$\leadsto i \frac{d}{dt} \begin{pmatrix} |P_L(t)\rangle \\ |P_H(t)\rangle \end{pmatrix} = \begin{pmatrix} m_L - \frac{i}{2} \Gamma_L & 0 \\ 0 & m_H - \frac{i}{2} \Gamma_H \end{pmatrix} \begin{pmatrix} |P_L(t)\rangle \\ |P_H(t)\rangle \end{pmatrix} \quad |$$

In order to study CP violation we can write the so-called 'Master Equations'.

Master Equations

Po

We already know the form of P^0 in terms of P_L and P_H :

$$|P^0(t)\rangle = \frac{1}{2p} \left\{ \underbrace{e^{-im_L t - \frac{\Gamma_L}{2} t}}_{P_L(t)} |P_L\rangle + \underbrace{e^{-im_H t - \frac{\Gamma_H}{2} t}}_{P_H(t)} |P_H\rangle \right\}$$

Let's now rewrite it using their definition in terms of P^0 and $\bar{P}^0$

$$|P^0(t)\rangle = \frac{1}{2p} \left\{ \begin{aligned} &e^{-im_L t - \frac{\Gamma_L}{2} t} (p|P^0\rangle - q|\bar{P}^0\rangle) + \\ &e^{-im_H t - \frac{\Gamma_H}{2} t} (p|P^0\rangle + q|\bar{P}^0\rangle) \end{aligned} \right\}$$

Rewriting the exponentials:

$$|P^0(t)\rangle = \frac{1}{2} \left(e^{-im_L t - \frac{\Gamma_L}{2} t} + e^{-im_H t - \frac{\Gamma_H}{2} t} \right) |P^0\rangle + \frac{q}{2p} \left(e^{-im_L t - \frac{\Gamma_L}{2} t} - e^{-im_H t - \frac{\Gamma_H}{2} t} \right) |\bar{P}^0\rangle$$

Let's now define $g_+(t)$ and $g_-(t)$

$$g_+(t) = \frac{1}{2} \left(e^{-im_L t - \frac{\Gamma_L}{2} t} + e^{-im_H t - \frac{\Gamma_H}{2} t} \right)$$

$$g_-(t) = +\frac{q}{2p} \left(e^{-im_L t - \frac{\Gamma_L}{2} t} - e^{-im_H t - \frac{\Gamma_H}{2} t} \right)$$

So

$$|P^0(t)\rangle = g_+(t) |P^0\rangle + \frac{q}{p} g_-(t) |\bar{P}^0\rangle$$

$$|\bar{P}^0(t)\rangle = g_+(t) |\bar{P}^0\rangle + \frac{p}{q} g_-(t) |P^0\rangle$$

$t=0$

CP violation in the Kaon System

ε is a complex number describing CP violation $\leadsto$ small

$\sim K_S$ associated with K_L in the beginning
 $\rightarrow$ Now we have to add K_S as well (ε)

$$|K_S\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}} (|K_1\rangle + \varepsilon |K_2\rangle)$$

$\sim K_L$ associated with K_2 in the beginning
 $\rightarrow$ Now we have to add K_S as well (ε)

$$|K_L\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}} (|K_2\rangle + \varepsilon |K_1\rangle)$$

Now we manipulate rewriting everything in function of $K^0, \bar{K}^0$ using the definition of K_1, K_2

$$|K_S\rangle = \frac{1}{\sqrt{2}\sqrt{1+|\varepsilon|^2}} \left[(|K^0\rangle + |\bar{K}^0\rangle) + \varepsilon (|K^0\rangle - |\bar{K}^0\rangle) \right]$$

$$|K_L\rangle = \frac{1}{\sqrt{2}\sqrt{1+|\varepsilon|^2}} \left[(|K^0\rangle - |\bar{K}^0\rangle) + \varepsilon (|K^0\rangle + |\bar{K}^0\rangle) \right]$$

Then

$$|K_S\rangle = \frac{1}{\sqrt{2}\sqrt{1+|\varepsilon|^2}} \left[(1+\varepsilon) |K^0\rangle + (1-\varepsilon) |\bar{K}^0\rangle \right]$$

$$|K_L\rangle = \frac{1}{\sqrt{2}\sqrt{1+|\varepsilon|^2}} \left[(1+\varepsilon) |K^0\rangle - (1-\varepsilon) |\bar{K}^0\rangle \right]$$

Defining

$$p = \frac{(1+\varepsilon)}{\sqrt{2}\sqrt{1+|\varepsilon|^2}} \quad ; \quad q = \frac{(1-\varepsilon)}{\sqrt{2}\sqrt{1+|\varepsilon|^2}} \quad ; \quad \frac{q}{p} = \frac{(1-\varepsilon)}{(1+\varepsilon)}$$

$$\text{with } p^2 + q^2 = 1$$

With this we can rewrite $|K^0\rangle, |\bar{K}^0\rangle$

$$|K^0\rangle = \frac{\sqrt{1+|\varepsilon|^2}}{\sqrt{2}(1+\varepsilon)} (|K_S\rangle + |K_L\rangle)$$

$$|\bar{K}^0\rangle = \frac{\sqrt{1+|\varepsilon|^2}}{\sqrt{2}(1-\varepsilon)} (|K_S\rangle - |K_L\rangle)$$

Experimental observation of CP violation

CP VIOLATION IN THE DECAY

$K_L \rightarrow \pi^+ e^- \bar{\nu}_e$
 Mass eigenstate

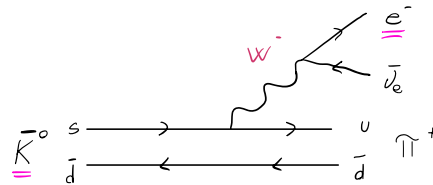
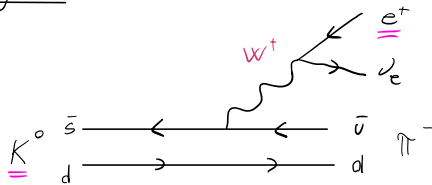
$K_L \rightarrow \pi^- e^+ \nu_e$

We should build an observable to measure CP violation

$$A_L = \frac{\Gamma(K_L \rightarrow \pi^- e^+ \nu_e) - \Gamma(K_L \rightarrow \pi^+ e^- \bar{\nu}_e)}{\Gamma(K_L \rightarrow \pi^- e^+ \nu_e) + \Gamma(K_L \rightarrow \pi^+ e^- \bar{\nu}_e)} \quad \text{no Related to CP violation}$$

however the decay has to be explained using Flavour eigenstates.

Decay diagrams



Let's write a decay rate

$$\begin{aligned} \underline{K_0} \quad \Gamma(K_L \rightarrow \pi^- e^+ \nu_e) &\propto |\langle K^0 | K_L \rangle|^2 = |1 + \varepsilon|^2 \\ \underline{\bar{K}_0} \quad \Gamma(K_L \rightarrow \pi^+ e^- \bar{\nu}_e) &\propto |\langle \bar{K}^0 | K_L \rangle|^2 = |1 - \varepsilon|^2 \end{aligned} \quad \left. \begin{array}{l} \text{Just those 2!} \\ K^0 \not\rightarrow \pi^+ e^- \bar{\nu}_e \\ \bar{K}^0 \not\rightarrow \pi^- e^+ \nu_e \end{array} \right\}$$

Now the asymmetry

$$A_L = \frac{|1 + \varepsilon|^2 - |1 - \varepsilon|^2}{|1 + \varepsilon|^2 + |1 - \varepsilon|^2} \quad \text{rewriting using } |1 - \varepsilon|^2 = (1 - \varepsilon)(1 - \varepsilon^*)$$

Now $\varepsilon = |\varepsilon| e^{-i\varphi}$

$$|1 - \varepsilon|^2 = (1 - \varepsilon)(1 - \varepsilon^*) = 1 - \varepsilon - \varepsilon^* + \varepsilon \varepsilon^* \sim 0$$

CP violation is small

$$\begin{aligned} -\varepsilon - \varepsilon^* &= -|\varepsilon| \cos \varphi - i|\varepsilon| \sin \varphi - |\varepsilon| \cos \varphi + i|\varepsilon| \sin \varphi = \\ &= -2 \operatorname{Re} \{\varepsilon\} \end{aligned}$$

$$|1 - \varepsilon^2| = 1 - 2 \operatorname{Re} \{\varepsilon\} + \mathcal{O}(\varepsilon^2)$$

$$|1 + \varepsilon^2| = 1 + 2 \operatorname{Re} \{\varepsilon\} + \mathcal{O}(\varepsilon^2)$$

Rewriting

$$A_L = \frac{|1 + \epsilon|^2 - |1 - \epsilon|^2}{|1 + \epsilon|^2 + |1 - \epsilon|^2} = \frac{1 + 2\text{Re}\{\epsilon\} - 1 - 2\text{Re}\{\epsilon\}}{1 + 2\text{Re}\{\epsilon\} + 1 - 2\text{Re}\{\epsilon\}} \simeq 2\text{Re}\{\epsilon\} = 2|\epsilon|\cos\varphi$$

DIRECT CP VIOLATION

^ Related only to the decay $\Rightarrow$ that's why direct

- If A_L is different from 0 then CP is violated

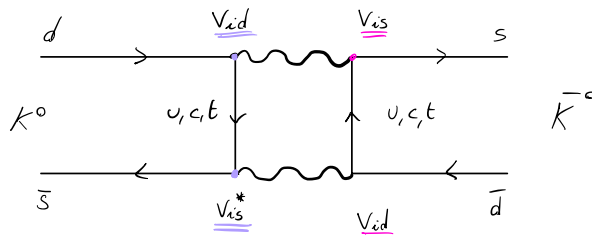
Measured value $A_L = (0.33 \pm 0.006) \cdot 10^{-2} \neq 0$



CP VIOLATION IN THE MIXING

Why the K mesons mix? The following diagrams has a probability different from 0.

Box diagram



Box DIAGRAM

$$K^0 \longleftrightarrow \bar{K}^0$$

$$\Delta S = 2$$

$$S = -1 \rightarrow S = +1$$

How to measure the CP violation in oscillation?

CPLEAR

Production

$$\begin{array}{ll} 1. p\bar{p} \rightarrow K^- \pi^+ K^0 & \begin{array}{ccc} p & \bar{p} & \rightarrow K^- \pi^+ \\ (uud) & (\bar{u}\bar{u}\bar{d}) & (\bar{s}u) \quad (\bar{u}\bar{d}) \end{array} \begin{array}{c} K^0 \\ (s\bar{d}) \end{array} \\ 2. p\bar{p} \rightarrow K^+ \pi^- \bar{K}^0 & \begin{array}{ccc} p & \bar{p} & \rightarrow K^+ \pi^- \\ (uud) & (\bar{u}\bar{u}\bar{d}) & (\bar{s}u) \quad (\bar{u}\bar{d}) \end{array} \begin{array}{c} \bar{K}^0 \\ (s\bar{d}) \end{array} \end{array}$$

Identification: the K_{con}^0 say the Flavour of the production

Decay

$$K^0 \rightarrow \pi^- e^+ \bar{\nu}_e$$

$$\bar{K}^0 \rightarrow \pi^+ e^- \bar{\nu}_e$$

~~$$K^0 \rightarrow \pi^+ e^- \bar{\nu}_e$$~~

Not Possible

~~$$\bar{K}^0 \rightarrow \pi^- e^+ \bar{\nu}_e$$~~

Again the decay is Flavor specific

$$K^0 \rightarrow \bar{K}^0 \rightarrow \pi^+ e^- \bar{\nu}_e$$

100% sure that there was MIXING

So we can tag the Flavour at the beginning and the Flavour at the end $\Rightarrow$ CREATE observable

	FLAVOUR AT PRODUCTION	FLAVOUR AT DECAY	
1.	K^0	$K^0 [e^+]$	NO MIXING
2.	$\bar{K}^0$	$\bar{K}^0 [e^-]$	NO MIXING
3.	K^0	$\bar{K}^0 [e^-]$	MIXING
4.	$\bar{K}^0$	$K^0 [e^+]$	MIXING

So

$$|K^0(t)\rangle = g_+(t) |K^0\rangle + \frac{q}{p} g_-(t) |\bar{K}^0\rangle$$

$$|\bar{K}^0(t)\rangle = g_+(t) |\bar{K}^0\rangle + \frac{p}{q} g_-(t) |K^0\rangle$$

$$P_2(K_{t=0}^0 \rightarrow K^0) = |\langle K^0 | K^0(t) \rangle|^2 = |g_+(t)|^2 \quad \text{NO MIXING}$$

$$P_2(\bar{K}^0 \rightarrow \bar{K}^0) = |\langle \bar{K}^0 | \bar{K}^0(t) \rangle|^2 = |g_+(t)|^2 \quad \text{NO MIXING}$$

$$P_3(K^0 \rightarrow \bar{K}^0) = |\langle \bar{K}^0 | K^0(t) \rangle|^2 = \left|\frac{q}{p}\right|^2 |g_-(t)|^2 \quad \text{MIXING}$$

$$P_4(\bar{K}_{t=0}^0 \rightarrow K^0) = |\langle K^0 | \bar{K}^0(t) \rangle|^2 = \left|\frac{p}{q}\right|^2 |g_-(t)|^2 \quad \text{MIXING}$$

EXERCISE 2

Calculate $|g_+(t)|$, g_- in terms of $\sinh, \cosh, \sin, \cos$

Solution:

$$P_1(t) = \frac{1}{4} (e^{-\Gamma_1 t} + e^{-\Gamma_2 t} + 2 e^{-\Gamma t} \cos \Delta m t)$$

$$P_2(t) = \frac{1}{4} (e^{-\Gamma_1 t} + e^{-\Gamma_2 t} + 2 e^{-\Gamma t} \cos \Delta m t)$$

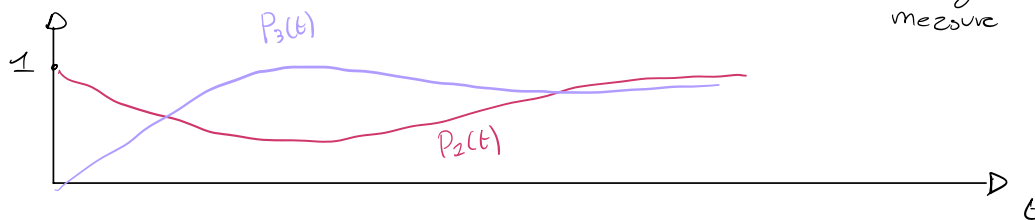
} NO MIXING

$$P_3(t) = \frac{1}{4} \left| \frac{q}{p} \right|^2 (e^{-\Gamma_1 t} + e^{-\Gamma_2 t} - 2 e^{-\Gamma t} \cos \Delta m t)$$

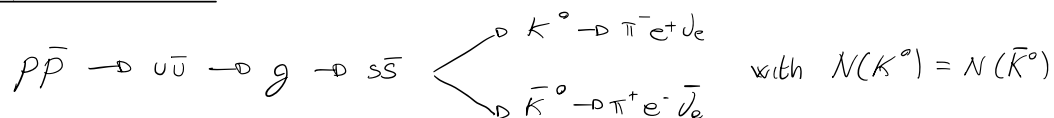
$$P_4(t) = \frac{1}{4} \left| \frac{p}{q} \right|^2 (e^{-\Gamma_1 t} + e^{-\Gamma_2 t} - 2 e^{-\Gamma t} \cos \Delta m t)$$

} MIXING

- the $\cos \Delta m t$ is common to the 3 probabilities $\gg$ IF you want to observe mixing you have to measure Δm



EXPERIMENTAL ASPECTS



we can count the rate, that is proportional to the probability

$$\begin{array}{l|l} \Gamma \underline{R}_+(t) \propto N P_2(K^0 \rightarrow K^0)(t) & \bar{R}_+(t) \propto N P_4(\bar{K}^0 \rightarrow K^0) \\ \text{\# collision} \swarrow & \\ \underline{R}_-(t) \propto N P_3(K^0 \rightarrow \bar{K}^0)(t) & \bar{R}_-(t) \propto N P_2(\bar{K}^0 \rightarrow \bar{K}^0) \end{array}$$

$\hookrightarrow$ identify the flavour at the decay

With those we build an asymmetry

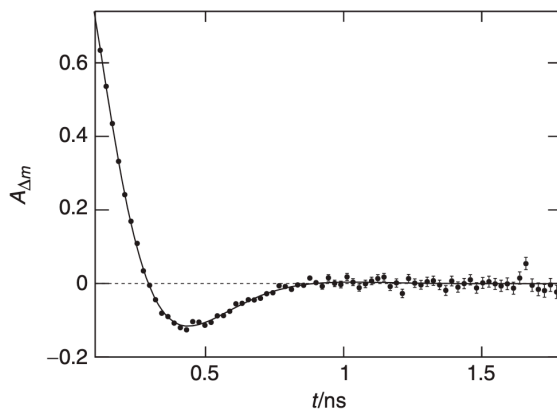
$$A(t) = \frac{[\overset{\text{NO MIX}}{R_+(t)} + \overset{\text{MIX}}{\bar{R}_-(t)}] - [\overset{\text{MIX}}{R_-(t)} + \bar{R}_+(t)]}{[\overset{\text{NO MIX}}{R_+(t)} + \bar{R}_-(t)] + [\overset{\text{MIX}}{R_-(t)} + \bar{R}_+(t)]}$$

Assuming that
CP is conserved
is better to

Assuming $\left| \frac{q}{p} \right| = \left| \frac{p}{q} \right| = 1$ NO ASYMMETRY

$$\begin{aligned} A(t) &= \frac{[P_2(t) + P_3(t)] - [P_1(t) + P_4(t)]}{[P_2(t) + P_3(t)] + [P_1(t) + P_4(t)]} \\ &= \frac{2 \frac{1}{4} (e^{-\Gamma_S t} + e^{-\Gamma_L t}) + 2 e^{-\Gamma t} \cos \Delta m t - 2 \frac{1}{4} \left(\frac{p}{q} \right)^2 (e^{-\Gamma_S t} + e^{-\Gamma_L t}) - 2 e^{-\Gamma t} \cos \Delta m t}{2 \frac{1}{4} (e^{-\Gamma_S t} + e^{-\Gamma_L t}) + 2 e^{-\Gamma t} \cos \Delta m t + 2 \frac{1}{4} \left(\frac{p}{q} \right)^2 (e^{-\Gamma_S t} + e^{-\Gamma_L t}) - 2 e^{-\Gamma t} \cos \Delta m t} \\ &= \frac{2 e^{-\Gamma t} \cos \Delta m t}{e^{-\Gamma_S t} + e^{-\Gamma_L t}} \end{aligned}$$

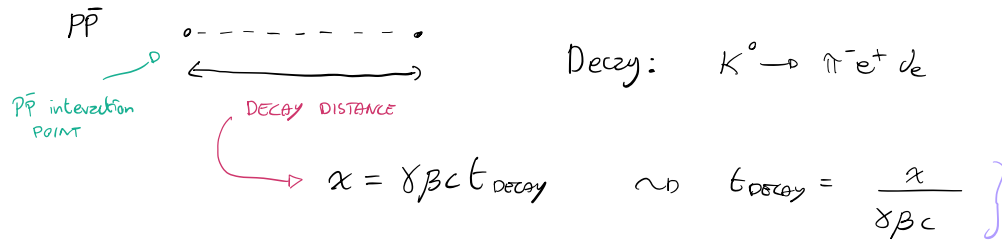
Plot $A(t)$ with real values



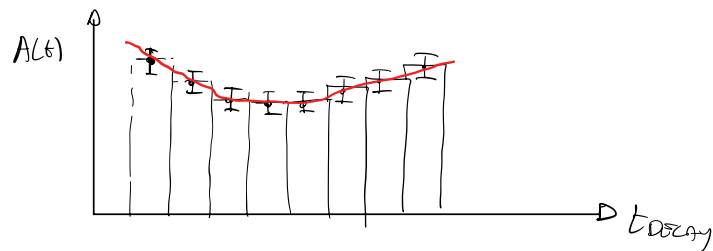
- $\Gamma = 1.12 \times 10^{-10} \text{ s}^{-1}$ (average decay width)
- $\Delta m = 3.485 \times 10^{-15} \text{ GeV}$ (mass difference)
- $\Gamma_S = 1.87 \times 10^{10} \text{ s}^{-1}$ (short-lived state decay width)
- $\Gamma_L = 7.30 \times 10^6 \text{ s}^{-1}$ (long-lived state decay width)

EXPERIMENTAL CONSIDERATIONS AND FIT

CLEAR



→ Count a lot of decays and build HISTOGRAMS

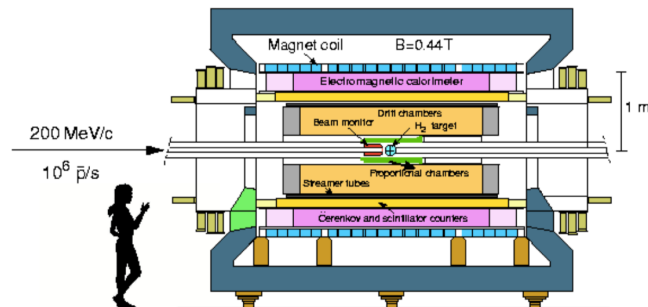


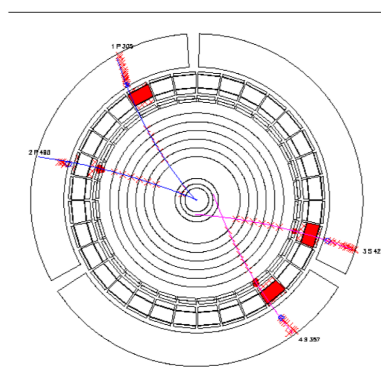
Perform a Fit by fitting the parameters τ_s, τ_c

→ You get the BEST FIT

AFTER THE FIT $\Delta m = (xxx \pm xxy)$

Experiment



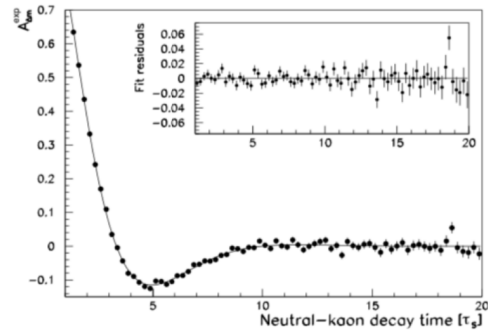
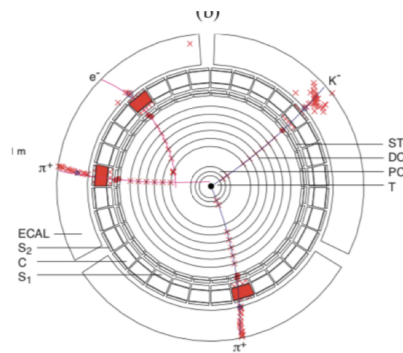


Kaon production at CPLEAR



The charge of the Kaon defines the flavour of the neutral Kaon at the time of the production.

This is the true measurement with the final result



$$A_{\Delta m}^{\text{theo}} = \frac{2e^{-\frac{\Gamma t}{2}} \cos(\Delta m \cdot t)}{e^{-\Gamma_s t} + e^{-\Gamma_L t}} \quad \text{Eur. Phys. J. C 22 (2001) 55-79}$$

$$\Delta m = (0.5295 \pm 0.0020_{\text{stat}} \pm 0.0003_{\text{syst}}) \times 10^{10} \text{ h/s},$$



Meson Decay

We consider also a Final state $f, \bar{f}$

~ We identify a neutral meson P^0 , but the formalism that we use is going to be the same also for K^0, D^0, B^0, B_s^0

$\Rightarrow P^0$ NEUTRAL MESON

$A_f \equiv \langle f | H | P^0 \rangle$ it's an amplitude.

~ We still have to specify the hamiltonian.

There are 4 amplitudes

$$A_f \equiv \langle f | H | P^0 \rangle$$

$$\bar{A}_f \equiv \langle f | H | \bar{P}^0 \rangle$$

$$A_{\bar{f}} \equiv \langle \bar{f} | H | P^0 \rangle$$

$$\bar{A}_{\bar{f}} \equiv \langle \bar{f} | H | \bar{P}^0 \rangle$$

~ H is the hamiltonian related to specific final state (ew, geo, ...)

Now we introduce some quantity that will be used later

$$\boxed{\lambda_f \equiv \frac{q}{p} \frac{\bar{A}_f}{A_f} \quad \bar{\lambda}_f = \frac{1}{\lambda_f} \quad \lambda_{\bar{f}} \equiv \frac{q}{p} \frac{\bar{A}_{\bar{f}}}{A_{\bar{f}}} \quad \bar{\lambda}_{\bar{f}} \equiv \frac{1}{\lambda_{\bar{f}}}}$$

Remembering that

$$|P^0(t)\rangle = g_+(t) |P^0\rangle + \frac{q}{p} g_-(t) |\bar{P}^0\rangle$$

$$|\bar{P}^0(t)\rangle = g_+(t) |\bar{P}^0\rangle + \frac{p}{q} g_-(t) |P^0\rangle$$

The probability that a certain decay will happen is $|A_f|^2 = |\langle f | H | P^0 \rangle|^2$

As before we can write 4 probabilities:

$$|A_f|^2 = |\langle f | H | P^0 \rangle|^2$$

$$|A_{\bar{f}}|^2 = |\langle \bar{f} | H | P^0 \rangle|^2$$

$$|\bar{A}_f|^2 = |\langle f | H | \bar{P}^0 \rangle|^2$$

$$|\bar{A}_{\bar{f}}|^2 = |\langle \bar{f} | H | \bar{P}^0 \rangle|^2$$

Time INTEGRATED AMPLITUDES (independent of time)

For the first calculation:

$$\begin{aligned} \Gamma_{P^0 \rightarrow f}(t) &= |\langle f | H | P^0(t) \rangle|^2 = \left| g_+(t) \overbrace{\langle f | H | P^0 \rangle}^{A_f} + \frac{q}{p} g_-(t) \overbrace{\langle f | H | \bar{P}^0 \rangle}^{\bar{A}_f} \right|^2 \\ &= |g_+(t)|^2 \cdot |A_f|^2 + \left| \frac{q}{p} \right|^2 |g_-(t)|^2 |\bar{A}_f|^2 + 2 \operatorname{Re} \left\{ \frac{q}{p} A_f^* \bar{A}_f g_+^*(t) g_-(t) \right\} \\ &= |A_f|^2 \left[|g_+(t)|^2 + \left| \frac{q}{p} \right|^2 \left| \frac{\bar{A}_f}{A_f} \right|^2 |g_-(t)|^2 + 2 \operatorname{Re} \left\{ \frac{q}{p} \frac{\bar{A}_f}{A_f} g_+^*(t) g_-(t) \right\} \right] \\ &= |A_f|^2 \left[|g_+(t)|^2 + \underbrace{\left| \frac{\bar{A}_f}{A_f} \right|^2}_{|\lambda_f|^2} |g_-(t)|^2 + 2 \operatorname{Re} \left\{ \underbrace{\frac{q}{p} \frac{\bar{A}_f}{A_f}}_{\lambda_f} g_+^*(t) g_-(t) \right\} \right] \end{aligned}$$

EXERCISE 3: compute the other $\neq \Gamma_s$

Solutions

$$\Gamma_{P^0 \rightarrow f}(t) = |A_f|^2 \left[|g_+(t)|^2 + |\lambda_f|^2 |g_-(t)|^2 + 2 \operatorname{Re} \{ \lambda_f g_+^*(t) g_-(t) \} \right]$$

$$\Gamma_{P^0 \rightarrow f}(t) = |A_f|^2 \left| \frac{p}{q} \right|^2 \left[|g_+(t)|^2 |\lambda_f|^2 + |g_-(t)|^2 + 2 \operatorname{Re} \{ \lambda_f^* g_+^*(t) g_-(t) \} \right]$$

$$\Gamma_{P^0 \rightarrow \bar{f}}(t) = |\bar{A}_f|^2 \left[|g_+(t)|^2 + |\lambda_{\bar{f}}|^2 |g_-(t)|^2 + 2 \operatorname{Re} \{ \lambda_{\bar{f}} g_+^*(t) g_-(t) \} \right]$$

$$\Gamma_{P^0 \rightarrow \bar{f}}(t) = |\bar{A}_f|^2 \left| \frac{p}{q} \right|^2 \left[|\lambda_{\bar{f}}|^2 |g_+(t)|^2 + |g_-(t)|^2 + 2 \operatorname{Re} \{ \lambda_{\bar{f}}^* g_+^*(t) g_-(t) \} \right]$$

Now let's consider

$$|g_{\pm}(t)|^2 = \frac{1}{4} (e^{-\frac{1}{2}\Gamma t} + e^{-\frac{1}{2}\Gamma t} + 2 e^{-\frac{1}{2}\Gamma t} \cos \Delta m t) \stackrel{\text{reversing}}{=} \frac{e^{-\frac{1}{2}\Gamma t}}{2} (\cosh \frac{\Delta \Gamma}{2} t \pm \cos \Delta m t)$$

$$g_+^*(t) g_-(t) = \stackrel{\text{reversing}}{=} \frac{e^{-\frac{1}{2}\Gamma t}}{2} (\sinh \frac{\Delta \Gamma}{2} t - i \sin \Delta m t)$$

$$\begin{aligned} \rightarrow \Gamma_{P^0 \rightarrow f}(t) &= |A_f|^2 \frac{e^{-\frac{1}{2}\Gamma t}}{2} \left[(\cosh \frac{\Delta \Gamma}{2} t + \cos \Delta m t) + |\lambda_f|^2 (\cosh \frac{\Delta \Gamma}{2} t - \cos \Delta m t) \right. \\ &\quad \left. + 2 \operatorname{Re} \{ \lambda_f \} \sinh \frac{\Delta \Gamma}{2} t + 2 \operatorname{Im} \{ \lambda_f \} \sin \Delta m t \right] \end{aligned}$$

Reverting it

$$\Gamma_{P^0 \rightarrow f}(t) = |A_f|^2 \frac{e^{-\frac{1}{2}\Gamma t}}{2} \left[(1 + |\lambda_f|^2) \cosh \frac{\Delta \Gamma}{2} t + (1 - |\lambda_f|^2) \cos \Delta m t + 2 \operatorname{Re} \{ \lambda_f \} \sinh \frac{\Delta \Gamma}{2} t + 2 \operatorname{Im} \{ \lambda_f \} \sin \Delta m t \right]$$

We are considering only the decay rate of $P^0, \bar{P}^0$ into f because there are interesting decays like $B^0 \rightarrow \pi^+ \pi^-$ $\leadsto$ They have $f = \bar{f}$ $\Rightarrow$ The only difference that we have to consider is

if we have different particle in the production $\leadsto$ There is ~~reason~~ to distinguish between particle and antiparticle: this is the reason why we only concentrate P^0 into final states!

Lets compute the other

$$\Gamma_{\bar{p}^0 \rightarrow f}(t) = |A_f|^2 \left| \frac{p}{q} \right|^2 \frac{e^{-\Gamma t}}{2} \left[|A_f|^2 \left(\cosh \frac{\Delta\Gamma}{2} t + \cos \Delta m t \right) + \cosh \frac{\Delta\Gamma}{2} t - \cos \Delta m t + 2 \operatorname{Re} \{ \lambda_f \} \sinh \frac{\Delta\Gamma}{2} t - \operatorname{Im} \{ \lambda_f \}^2 \sin \Delta m t \right]$$

rearranging

$$\Gamma_{\bar{p}^0 \rightarrow f}(t) = |A_f|^2 \left| \frac{p}{q} \right|^2 \frac{e^{-\Gamma t}}{2} \left[(1 + |\lambda_f|^2) \cosh \frac{\Delta\Gamma}{2} t - (1 - |\lambda_f|^2) \cos \Delta m t + 2 \operatorname{Re} \{ \lambda_f \} \sinh \frac{\Delta\Gamma}{2} t - 2 \operatorname{Im} \{ \lambda_f \} \sin \Delta m t \right]$$

Everything that we write is going to depend on final states

$$\Delta f = \frac{2 \operatorname{Re} \{ \lambda_f \}}{1 + |\lambda_f|^2}$$

$$C_f = \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2}$$

$$S_f = \frac{2 \operatorname{Im} \{ \lambda_f \}}{1 + |\lambda_f|^2}$$

$$\Gamma_{p^0 \rightarrow f}(t) = |A_f|^2 (1 + |\lambda_f|^2) \frac{e^{-\Gamma t}}{2} \left[\cosh \frac{\Delta\Gamma}{2} t + C_f \cos \Delta m t + \Delta f \sinh \frac{\Delta\Gamma}{2} t + S_f \sin \Delta m t \right]$$

$$\Gamma_{\bar{p}^0 \rightarrow f}(t) = |A_f|^2 (1 + |\lambda_f|^2) \left| \frac{p}{q} \right|^2 \frac{e^{-\Gamma t}}{2} \left[\cosh \frac{\Delta\Gamma}{2} t - C_f \cos \Delta m t + \Delta f \sinh \frac{\Delta\Gamma}{2} t - S_f \sin \Delta m t \right]$$

- the $\Delta f, C_f, S_f$ are related to the physics that is in the decay
- cos, sin functions are related to the mixing

EXERCISE: compute the other 2

Classification of CP violation

1. CP VIOLATION IN THE DECAY: (TIME - INDEPENDENT)

$$\Gamma_{P^0 \rightarrow f} \neq \Gamma_{\bar{P}^0 \rightarrow f} \quad \leadsto \quad \left| \frac{\bar{A}_f}{A_f} \right| \neq 1$$

$$\text{IF } f = \bar{f} \quad \Gamma_{P^0 \rightarrow f} \neq \Gamma_{\bar{P}^0 \rightarrow f} \quad \leadsto \quad \left| \frac{\bar{A}_f}{A_f} \right| \neq 1$$

EXAMPLE $B^0 \rightarrow K \bar{\pi}^+$ ($f \neq \bar{f}$)

$$A_{CP} = \frac{\Gamma_{B^0 \rightarrow K^+ K^-} - \Gamma_{B^0 \rightarrow K^- \bar{K}^+}}{\Gamma_{B^0 \rightarrow K^+ K^-} + \Gamma_{B^0 \rightarrow K^- \bar{K}^+}} \neq 0 \quad \Rightarrow \quad \text{CP violation}$$

~ also named DIRECT CP violation (in the decay $\rightarrow$ only depends on the final state)

2. CP VIOLATION IN THE MIXING

$$\text{Prob}(P^0 \rightarrow \bar{P}^0) \neq \text{Prob}(\bar{P}^0 \rightarrow P^0)$$

/ can study

$$A = \frac{\text{Prob}(P^0 \rightarrow \bar{P}^0) - \text{Prob}(\bar{P}^0 \rightarrow P^0)}{\text{Prob}(P^0 \rightarrow \bar{P}^0) + \text{Prob}(\bar{P}^0 \rightarrow P^0)} \neq 0$$

we are considering

$$|P^0(t)\rangle = g_+(t) |P^0\rangle + \frac{q}{p} g_-(t) |\bar{P}^0\rangle$$

$$|\bar{P}^0(t)\rangle = g_+(t) |\bar{P}^0\rangle + \frac{p}{q} g_-(t) |P^0\rangle$$

Let's now write

$$\text{Prob}(\bar{P}^0 \rightarrow P^0) = |\langle P^0 | \bar{P}^0(t) \rangle|^2 = \left| \frac{q}{p} \right|^2 |g_-(t)|^2$$

$$\text{Prob}(P^0 \rightarrow \bar{P}^0) = |\langle \bar{P}^0 | P^0(t) \rangle|^2 = \left| \frac{p}{q} \right|^2 |g_-(t)|^2$$

If we build the asymmetry we will have that the A doesn't depend on time.

Lets see

$$A = \frac{\left| \frac{q}{p} \right|^2 \cancel{|g(t)|^2} - \left| \frac{p}{q} \right|^2 \cancel{|g(t)|^2}}{\left| \frac{q}{p} \right|^2 \cancel{|g(t)|^2} + \left| \frac{p}{q} \right|^2 \cancel{|g(t)|^2}} = \frac{\left| \frac{q}{p} \right|^2 - \left| \frac{p}{q} \right|^2}{\left| \frac{q}{p} \right|^2 + \left| \frac{p}{q} \right|^2}$$

$\Rightarrow$ To have CP violation in the mixing $A \neq 0$ $\left| \frac{q}{p} \right|$ or $\left| \frac{p}{q} \right| \neq 1$

In the kaon system CP violation in mixing has been observed

$$|\epsilon| = (2.228 \pm 0.011) \cdot 10^{-3}$$

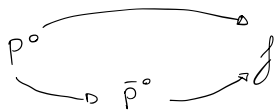
$$\left| \frac{q}{p} \right| = \left| \frac{1-\epsilon}{1+\epsilon} \right| = 0.995522 \pm 0.000024$$

In the case of B^0, B_s^0 NO EVIDENCE of CP violation in the mixing

$$\left. \begin{array}{l} B^0: \quad \left| \frac{q}{p} \right| = 1.0010 \pm 0.0008 \\ B_s^0: \quad \left| \frac{q}{p} \right| = 1.0003 \pm 0.0014 \end{array} \right\} \text{Compatible with 1}$$

3. CP VIOLATION IN THE INTERFERENCE BETWEEN MIXING AND DECAY

$$f = \bar{f}$$



As amplitude that is the sum of the 2 parts squared $\rightarrow$ INTERFERENCE

$$\Gamma_{P^0 \sim \bar{P}^0 \rightarrow f}(t) \neq \Gamma_{\bar{P}^0 \sim P^0 \rightarrow f}(t)$$

$$\left| \frac{q}{p} \right| = 1$$

THIS IS TIME DEPENDENT ANALYSIS

We build the

$$A_{CP}(t) = \frac{\Gamma_{P^0 \rightarrow f}(t) - \Gamma_{\bar{P}^0 \rightarrow f}(t)}{\Gamma_{P^0 \rightarrow f}(t) + \Gamma_{\bar{P}^0 \rightarrow f}(t)}$$

$$N(t) = C_f \cos \Delta m t + S_f \sin \Delta m t$$

$$D(t) = \cosh \frac{\Delta \Gamma}{2} t + \Delta f \sinh \frac{\Delta \Gamma}{2} t \quad \left. \vphantom{\cosh} \right\} A_{CP} = \frac{C_f \cos \Delta m t + S_f \sin \Delta m t}{\cosh \frac{\Delta \Gamma}{2} t + \Delta f \sinh \frac{\Delta \Gamma}{2} t}$$

$\Rightarrow$ To have CP violation

$C_f \neq 0$ and/or $S_f \neq 0$

Remembering the definition of C_f and S_f , we have 3 cases

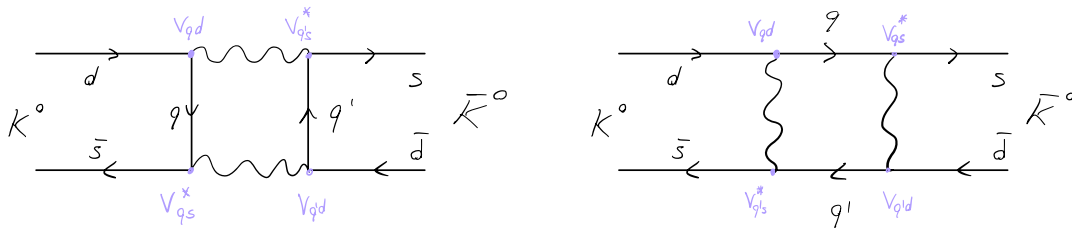
$$\left. \begin{array}{l} 1. \text{ No CP violation } \Rightarrow |A_f| = |\bar{A}_f| \\ 2. \text{ No CP violation } \Rightarrow \left| \frac{q}{p} \right| = 1 \end{array} \right\} C_f = 0$$

3. CP violation can still be observed if $S_f \neq 0$

$$\text{Im} \left\{ \frac{q}{p} \frac{\bar{A}_f}{A_f} \right\}$$

Interpretation of neutral mixing

Kaon System



$$q = \{u, c, t\}, \quad q' = \{u, c, t\} \quad \text{and the matrix element } \mathcal{M}_{qq'}$$

- the * on the CKM matrix when UP $\rightarrow$ DOWN like, opposite no star

$\rightarrow$ The diagrams are 2 but conceptually the calculation are the same

Let's see (Feynman theory for W propagator)

$$\mathcal{M}_{u,u} \propto G_F^2 (V_{ud} V_{us}^*)^2 \cdot \frac{1}{(k^2 - m_u^2)^2} \propto 1^2$$

$$\mathcal{M}_{c,c} \propto G_F^2 (V_{cd} V_{cs}^*)^2 \cdot \frac{1}{(k^2 - m_c^2)^2} \propto \underline{\underline{1^2}}$$

$$\mathcal{M}_{t,t} \propto G_F^2 (V_{td} V_{ts}^*)^2 \cdot \frac{1}{(k^2 - m_t^2)^2} \propto 1^{10}$$

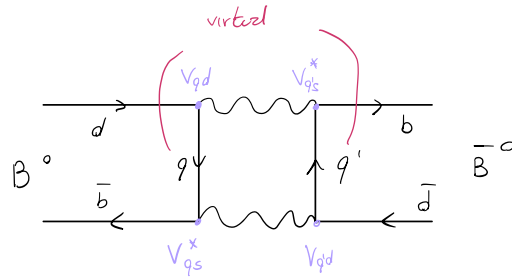
$$\mathcal{M}_{u,c} \propto G_F^2 (V_{ud} V_{us}^*) (V_{cd} V_{cs}^*) \frac{1}{(k^2 - m_u^2)} \frac{1}{(k^2 - m_c^2)} \propto 1^2$$

$$\mathcal{M}_{c,t} \propto G_F^2 (V_{cd} V_{cs}^*) (V_{td} V_{ts}^*) \frac{1}{(k^2 - m_c^2)} \frac{1}{(k^2 - m_t^2)} \propto 1^6$$

$$\mathcal{M}_{u,t} \propto G_F^2 (V_{ud} V_{us}^*) (V_{td} V_{ts}^*) \frac{1}{(k^2 - m_u^2)} \frac{1}{(k^2 - m_t^2)} \propto 1^6$$

As you can see the $\mathcal{M}_{cc}$ is the dominant contribution

B^0 system



$$q = \{u, c, t\}$$

$$q' = \{u, c, t\}$$

$$\text{Let's see } \mathcal{M}_{u,u} \propto G_F^2 (V_{ud} V_{ub}^*)^2 \cdot \frac{1}{(k^2 - m_u^2)^2} \propto \tau^6$$

$$\mathcal{M}_{c,c} \propto G_F^2 (V_{cd} V_{cb}^*)^2 \cdot \frac{1}{(k^2 - m_c^2)^2} \propto \tau^6$$

$$\mathcal{M}_{t,t} \propto G_F^2 (V_{td} V_{tb}^*)^2 \cdot \frac{1}{(k^2 - m_t^2)^2} \propto \tau^6$$

More suppressed
with respect to
the k_{Zon}

$\Downarrow$

The propagator is
more important

The $\mathcal{M}_{tt}$ is the DOMINANT one!

$$\mathcal{M}_{u,c} \propto G_F^2 (V_{ud} V_{ub}^*) (V_{cd} V_{cb}^*) \frac{1}{(k^2 - m_u^2)} \frac{1}{(k^2 - m_c^2)} \propto \tau^6$$

$$\mathcal{M}_{c,t} \propto G_F^2 (V_{cd} V_{cb}^*) (V_{td} V_{tb}^*) \frac{1}{(k^2 - m_c^2)} \frac{1}{(k^2 - m_t^2)} \propto \tau^6$$

$$\mathcal{M}_{u,t} \propto G_F^2 (V_{ud} V_{ub}^*) (V_{td} V_{tb}^*) \frac{1}{(k^2 - m_u^2)} \frac{1}{(k^2 - m_t^2)} \propto \tau^6$$

~ This was done before the discovery of the $m_{\text{ZSS}} \Rightarrow$ Use this for predicting Δm_q .

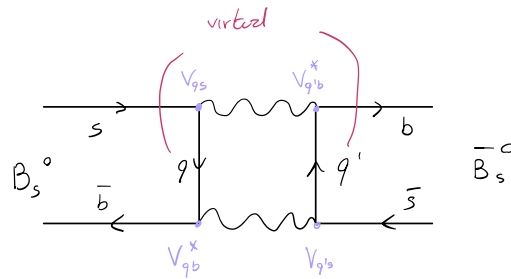
$$\text{If } \Delta m_K = \frac{G_F^2 m_W^2}{6\pi^2} \eta_{\text{aco}} B_K \int_K^2 m_K S_0\left(\frac{m_c^2}{m_W^2}\right) |V_{cd} V_{cs}|^2 \quad \underline{=}$$

$\sim m_K$ m_{ZSS} of K_{ZON} , m_W mass of W boson

$$S_0\left(\frac{m_c^2}{m_W^2}\right) \rightarrow S_0(x_q) \quad \text{with} \quad x_q = \frac{m_c^2}{m_W^2} \quad \text{INAMI-LIN FACTOR}$$

$$\leadsto m_W^2 S_0\left(\frac{m_c^2}{m_W^2}\right) \sim m_c^2 \Rightarrow \Delta m_q = \frac{G_F^2 m_W^2}{6\pi^2} \eta_{\text{aco}} \int_B^2 B_{Bq} m_B S_0\left(\frac{m_c^2}{m_W^2}\right) |V_{tq} V_{tb}|^2$$

B_s system



Again as before

$$\mathcal{M}_{cc} \propto (V_{us} V_{ub}^*)^2 \propto 1^6$$

$$\mathcal{M}_{cc} \propto (V_{cs} V_{cb}^*)^2 \propto 1^6$$

$$\mathcal{M}_{tb} \propto (V_{ts} V_{tb}^*)^2 \propto 1^6$$

$$\mathcal{M}_{us} \propto (V_{us} V_{ub}^* V_{cs} V_{cb}^*) \propto 1^6$$

$$\mathcal{M}_{ut} \propto (V_{us} V_{ub}^* V_{ts} V_{tb}^*) \propto 1^6$$

$$\mathcal{M}_{ut} \propto (V_{cs} V_{cb}^* V_{ts} V_{tb}^*) \propto \underline{1^4}$$

DOMINANT

$\mathcal{M}_{ut}$ is the dominant contribution

NOTE:

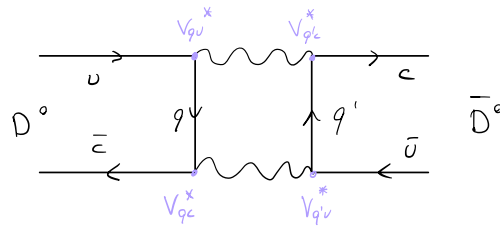
Δm_{B_s} is 10 times bigger than Δm_B

→ Since Δm is like a measurement of how fast oscillation happens so B_s is oscillating faster → we will see this better after.

In the case of $D^0(cu)$:

⇒ There is something special: different quarks in the propagator.

D^0 system



Down-type: different phenomenology

$$q = \{d, s, b\}$$

$$q' = \{d, s, b\}$$

Again as before

$$\mathcal{M}_{dd} \propto (V_{ud}^* V_{cd})^2 \propto 1^2$$

$$\mathcal{M}_{ss} \propto (V_{us}^* V_{cs})^2 \propto \underline{1}^2 \quad - \quad \text{DOMINANT}$$

$$\mathcal{M}_{bb} \propto (V_{ub}^* V_{cb})^2 \propto 1^{10}$$

$$\mathcal{M}_{ds} \propto (V_{ud}^* V_{cd} V_{us}^* V_{cs}) \propto 1^2$$

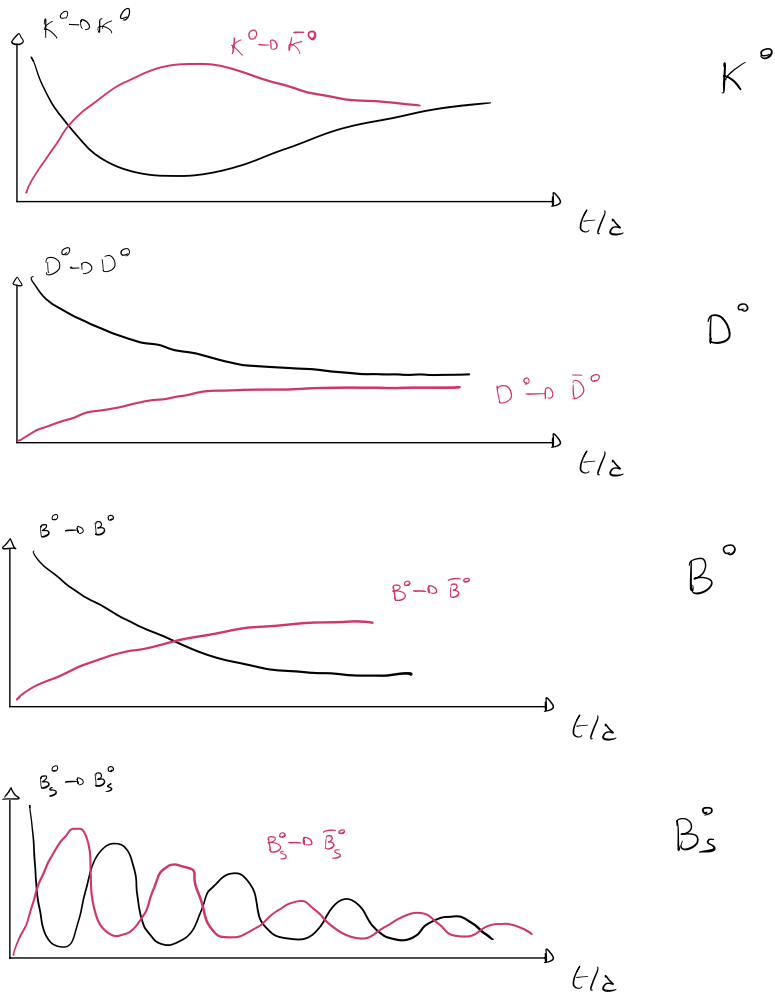
$$\mathcal{M}_{db} \propto (V_{ud}^* V_{cd} V_{ub}^* V_{cb}) \propto 1^6$$

$$\mathcal{M}_{sb} \propto (V_{us}^* V_{cs} V_{ub}^* V_{cb}) \propto 1^6$$

In the end if we consider all the calculation that we have done

$$\begin{aligned} K^0 \rightarrow \Delta m_K &\propto 1^2 m_c^2 \\ B^0 \rightarrow \Delta m_d &\propto 1^6 m_b^2 \\ B_s^0 \rightarrow \Delta m_s &\propto 1^4 m_t^2 \\ D^0 \rightarrow \Delta m &\propto 1^2 m_s^2 + \dots \end{aligned}$$

We can represent graphically the oscillation in t/τ of the different system to show how fast and how slow they are compared to each other.



Now let's express numerically this graphical representation by defining 2 quantities:

$$x = \frac{\Delta m}{\tau} \quad \text{and} \quad y = \frac{\Delta \Gamma}{2\tau}$$

	$\tau = 1/\Gamma [s]$	$\Delta m [s]$	x	y
K^0	$0.26 \cdot 10^{-9}$	$5.29 \cdot 10^{-9}$	0.4077	-1
D^0	$0.41 \cdot 10^{-12}$	$0.024 \cdot 10^{-12}$	0.0057	0.0078
B^0	$1.53 \cdot 10^{-12}$	$0.507 \cdot 10^{-12}$	0.78	0.0015
B_s^0	$1.47 \cdot 10^{-12}$	$17.77 \cdot 10^{-12}$	26.1	0.06

CP violation in the B system

We'll talk about Golden decay

$$B^0 \rightarrow \pi/\gamma K_S^0$$

related to the
B-angle of
the unitarity triangle

Let's define the observable for this decay.

We have to note that $(\pi/\gamma K_S^0)$ is the CP eigenstate we know that
if CP eigenstate $A_f = A_{\bar{f}}$ because $f = \bar{f}$, and

$$CP|f\rangle = \pm \eta_f |f\rangle$$

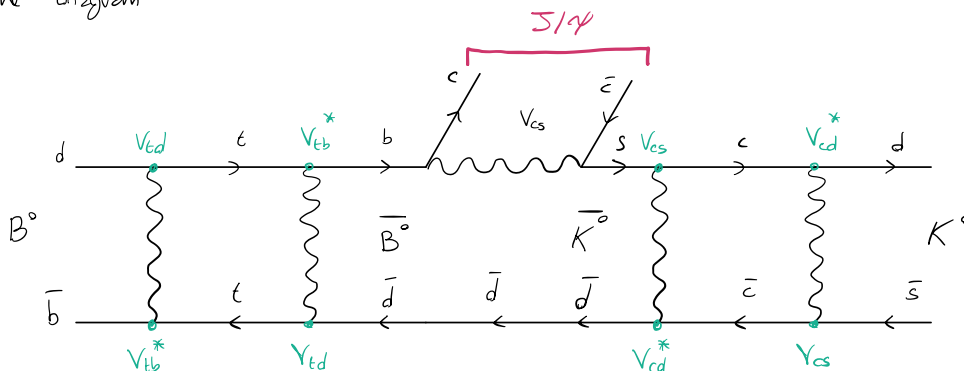
$$\eta_f = \pm 1 \begin{cases} K_S^0 \\ K_L^0 \end{cases}$$

We have 2 master equations $\Gamma_{B^0 \rightarrow f}(t)$ and $\Gamma_{\bar{B}^0 \rightarrow f}(t)$

With

$$A_{CP}(t) = \frac{\Gamma_{B^0 \rightarrow f}(t) - \Gamma_{\bar{B}^0 \rightarrow f}(t)}{\Gamma_{B^0 \rightarrow f}(t) + \Gamma_{\bar{B}^0 \rightarrow f}(t)}$$

Here the diagram



And we could have

$$\left. \begin{array}{l} \bar{B}^0 \rightarrow \pi/\gamma \bar{K}^0 \\ B^0 \rightarrow \pi/\gamma K^0 \end{array} \right\} B^0 \rightarrow \pi/\gamma K_S^0$$

Let's write K_s and K_L

$$|K_s\rangle = p|K^0\rangle + q|\bar{K}^0\rangle$$

$$|K_L\rangle = p|K^0\rangle - q|\bar{K}^0\rangle$$

In the B^0 system

$$\Delta\Gamma \sim 0, \quad |A_f| = |\bar{A}_f| \rightarrow |\lambda_f| = 1; \quad \left|\frac{P}{Q}\right| = 1 \quad \Rightarrow C_f = 0$$

And

$$A_{CP}(t) = \frac{\text{Im}|\lambda_f|}{2} \sin \Delta m(t)$$

$$\leadsto \lambda_f = \left(\frac{q}{p}\right)_{B^0} \left(\underbrace{\eta_{S/\psi K_s^0}}_{=1} \frac{\bar{A}_{S/\psi K^0}}{A_{S/\psi K^0}} \right) \left(\frac{P}{Q}\right)_{K^0}$$

$$\eta_{S/\psi K_L^0} = -1$$

$$\eta_{S/\psi K_S^0} = +1$$

Now let's consider this

$B^0 \rightarrow$ spin 0 particle

$S/\psi \rightarrow$ spin 1 particle

CP even

$\left. \begin{array}{l} \text{CP even} \\ \text{CP even} \end{array} \right\} l=1$

$$(-1)^l = -1$$

$K_S^0 \rightarrow$ spin 0 particle

CP even

$K_L^0 \rightarrow$ spin 0 particle

CP odd

$\left. \begin{array}{l} \text{CP even} \\ \text{CP odd} \end{array} \right\} \rightarrow \eta_{S/\psi K_L^0} = +1$

$$\eta_{S/\psi K_L^0} = +1$$

With those considerations let's consider some term of λ_f

$$1. \quad \left(\frac{q}{p}\right)_{B^0} = \sqrt{\frac{M_{12}^*}{M_{22}}} = \frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*} \leadsto M_{12} \propto (V_{tb} V_{td}^*)^2$$

$$2. \quad \frac{\bar{A}_{S/\psi K^0}}{A_{S/\psi K^0}} = \frac{V_{cb} V_{cs}^*}{V_{cb}^* V_{cs}}$$

$$3. \quad \left(\frac{P}{Q}\right)_{K^0} = \sqrt{\frac{M_{12}}{M_{22}^*}} = \frac{V_{cs} V_{cd}^*}{V_{cs}^* V_{cd}} \leadsto M_{12} \propto (V_{cs} V_{cd}^*)^2$$

Putting everything together

$$\lambda_{S/r K_S} = - \frac{\underbrace{V_{tb}^* V_{td}}_{\substack{\downarrow \\ \lambda_{S/r K_S}}}}{\underbrace{V_{tb} V_{ts}^*}} \cdot \frac{V_{cb} V_{cs}^*}{\underbrace{V_{cb}^* V_{cs}}} \cdot \frac{V_{cs} V_{cd}^*}{\underbrace{V_{cs}^* V_{cd}}} = \frac{\overbrace{V_{tb}^* V_{td}^*}^{|V_{td}| e^{-i\beta}}}{\underbrace{V_{tb} V_{td}^*}_{|V_{td}| e^{+i\beta}}} \cdot \frac{V_{cb} V_{cd}^*}{V_{cb}^* V_{cd}}$$

In the end

$$\lambda_{S/r K_S} = - |\lambda_{S/r K_S}| e^{-i2\beta}$$

$$- \text{Im}\{\lambda_{S/r K_S}\} = + \sin 2\beta$$

$$A_{CP}(t) = \underbrace{\sin 2\beta}_{\substack{\text{Amplitude of} \\ \sin \Delta m}} \sin \Delta m t$$

Observation of CP violation

Let's consider

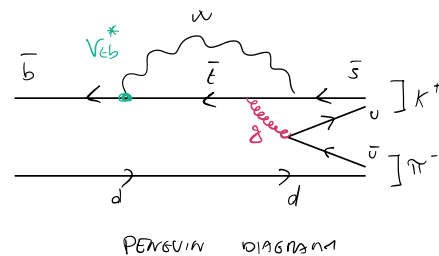
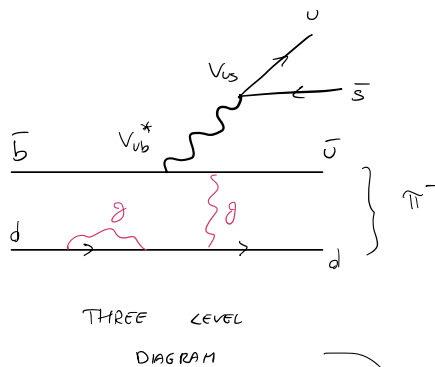
$$B^0 \rightarrow K^+ \pi^- \quad \bar{B}^0 \rightarrow K^- \pi^+ \quad f \neq \bar{f}$$

if we measure $|A_f| \neq |\bar{A}_f| \sim \text{CP violation}$

FORBIDDEN

$$B^0 \rightarrow K^- \pi^+ \\ \bar{B}^0 \rightarrow K^+ \pi^-$$

Let's see the diagrams



Same order of
magnitude

For the three level:

$$\begin{aligned} \text{THREE LEVEL} \quad A_T &= |A_T| e^{i\delta_T} e^{i\varphi_T} \\ \text{THREE LEVEL} \quad \bar{A}_T &= |A_T| e^{i\delta_T} e^{-i\varphi_T} \end{aligned} \quad \left. \begin{array}{l} \text{Strong phase} \\ \text{Weak phase} \end{array} \right\} \text{CP change only the weak phase}$$

For the Penguin level:

$$\begin{aligned} \text{PENGUIN LEVEL} \quad A_P &= |A_P| e^{i\delta_P} e^{i\varphi_P} \\ \text{PENGUIN LEVEL} \quad \bar{A}_P &= |A_P| e^{i\delta_P} e^{-i\varphi_P} \end{aligned}$$

So the total amplitude For $B^0 \rightarrow K^+ \pi^-$: $BR(B^0 \rightarrow K^+ \pi^-)$

$$\begin{aligned}
 |A_T + A_P|^2 &= (|A_T| e^{i\delta_T} e^{i\varphi_T} + |A_P| e^{i\delta_P} e^{i\varphi_P}) + \\
 &\quad (|A_T| e^{-i\delta_T} e^{-i\varphi_T} + |A_P| e^{-i\delta_P} e^{-i\varphi_P}) \\
 &= |A_T|^2 + |A_P|^2 + |A_T||A_P| e^{i(\delta_T - \delta_P)} e^{i(\varphi_T - \varphi_P)} \\
 &\quad + |A_T||A_P| e^{-i(\delta_T - \delta_P)} e^{-i(\varphi_T - \varphi_P)} \\
 &= |A_T|^2 + |A_P|^2 + |A_T||A_P| \left[e^{i(\delta_T - \delta_P)} e^{i(\varphi_T - \varphi_P)} + e^{-i(\delta_T - \delta_P)} e^{-i(\varphi_T - \varphi_P)} \right] \\
 &= |A_T|^2 + |A_P|^2 + |A_T||A_P| \left[\cos(\delta_T - \delta_P) + \cos(\varphi_T - \varphi_P) \right] \\
 &= |A_T|^2 + |A_P|^2 + |A_T||A_P| \left[\cos(\delta_T - \delta_P) \cos(\varphi_T - \varphi_P) - \sin(\delta_T - \delta_P) \sin(\varphi_T - \varphi_P) \right]
 \end{aligned}$$

We can consider

$$\bar{B}^0 \rightarrow K^- \pi^+$$

$$|\bar{A}_T + \bar{A}_P| = |A_T|^2 + |A_P|^2 + 2|A_T||A_P| \left[\cos(\delta_T - \delta_P) \cos(\varphi_T - \varphi_P) + \sin(\delta_T - \delta_P) \sin(\varphi_T - \varphi_P) \right]$$

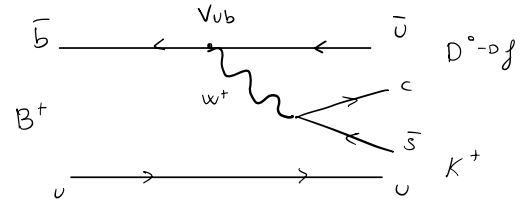
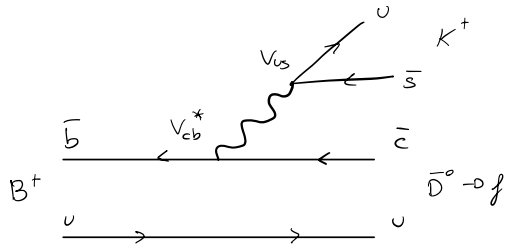
Building the asymmetry

$$A_{CP} = \frac{|\bar{A}_T + \bar{A}_P|^2 - |A_T + A_P|^2}{|\bar{A}_T + \bar{A}_P|^2 + |A_T + A_P|^2} = \frac{4|A_T||A_P| \sin(\delta_T - \delta_P) \sin(\varphi_T - \varphi_P)}{2|A_T|^2 + 2|A_P|^2 + 4|A_T||A_P| \cos(\delta_T - \delta_P) \cos(\varphi_T - \varphi_P)}$$

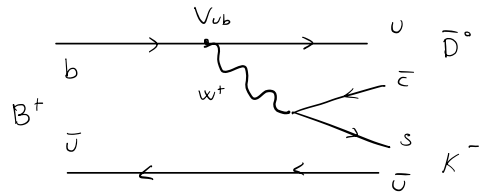
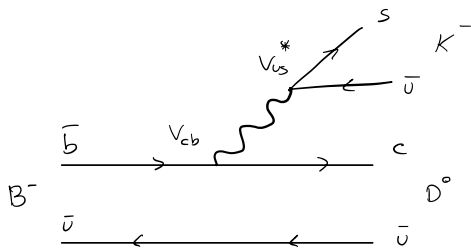
ONLY when the interference between PenguIn and ThreE is different from zero you can observe CP violation

$$\rightarrow \delta_T - \delta_P \neq 0, \quad \varphi_T - \varphi_P \neq 0$$

$$B^{\pm} \rightarrow \bar{D}^0 K^{\pm} \rightsquigarrow B^+ \rightarrow \bar{D}^0 K^+ \parallel B^- \rightarrow \bar{D}^0 K^-$$



2nd $A(B^+ \rightarrow \bar{D}^0 K^+) = |A_1| e^{i\delta_2} + |A_2| e^{i\delta} e^{i\delta_2}$



2nd $A(B^- \rightarrow \bar{D}^0 K^-) = |A_1| e^{i\delta_2} + |A_2| e^{i\delta_2} e^{-i\delta}$

- $A(B^+ \rightarrow \bar{D}^0 K^+) = |A_1|^2 + |A_2|^2 + 2|A_1||A_2| \sin(\delta_2 - \delta_1) \sin \delta$

- $A(B^- \rightarrow \bar{D}^0 K^-) = |A_1|^2 + |A_2|^2 + 2|A_1||A_2| \sin(\delta_2 - \delta_1) \sin \delta$

$$\Rightarrow A_{CP} = \frac{|A(B^+)|^2 - |A(B^-)|^2}{|A(B^+)|^2 + |A(B^-)|^2} = \frac{2|A_1||A_2| \sin(\delta_2 - \delta_1) \sin \delta}{2|A_1|^2 + 2|A_2|^2}$$

Charm Physics

We first have to define some variables:

$$x = \frac{\Delta m_D}{\Gamma_D} \quad y = \frac{\Delta \Gamma_D}{2 \Gamma_D}$$

then $\Delta \Gamma_D = \Gamma_1 - \Gamma_2$ and $\Gamma_D = \frac{\Gamma_1 + \Gamma_2}{2}$ $\Delta m_D = m_1 - m_2$

$$x \sim 1\% \quad y \sim 1\% \quad \text{CP EIGENSTATES} \quad \ell = \bar{\ell}$$

The only difference with previous calculation is that x, y are small

$$\rightarrow \underline{\Gamma_{D \rightarrow f}}(t) = |A_f|^2 \frac{e^{-\Gamma_D t}}{2} \left[(1 + |x_f|^2) \cosh \frac{\Delta \Gamma_D}{2} t + (1 - |x_f|^2) \cos \Delta m_D t + 2 \operatorname{Re}\{x_f\} \sinh \frac{\Delta \Gamma_D}{2} t + 2 \operatorname{Im}\{x_f\} \sin \Delta m_D t \right]$$

$$\rightarrow \underline{\Gamma_{\bar{D} \rightarrow f}}(t) = |A_f|^2 \frac{e^{-\Gamma_D t}}{2} \left| \frac{P}{Q} \right|^2 \left[(1 + |x_f|^2) \cos \frac{\Delta \Gamma_D}{2} t - (1 - |x_f|^2) \cos \Delta m_D t + 2 \operatorname{Re}\{x_f\} \sinh \frac{\Delta \Gamma_D}{2} t - \operatorname{Im}\{x_f\} \sin \Delta m_D t \right]$$

Let's now rewrite

$$\Delta m_D = x \cdot \Gamma_D \quad \text{and} \quad \frac{\Delta \Gamma_D}{2} = y \cdot \Gamma_D$$

$$\leadsto x_f = \frac{q}{p} \frac{\bar{A}_f}{A_f} = -\eta_f^{\text{CP}} \left| \frac{\bar{A}_f}{A_f} \right| \left| \frac{q}{p} \right| e^{i\phi}$$

And let's make some approximation

$$\cosh(y \Gamma_D t) \sim 1; \quad \cos(x \Gamma_D t) \sim 1; \quad \sin(x \Gamma_D t) \sim x \Gamma_D t; \quad \sinh(y \Gamma_D t) \sim y \cdot \Gamma_D t$$

Now let's rewrite

$$\underline{\hat{T}_{D \rightarrow D_f}(t)} = |A_f|^2 \frac{e^{-\Gamma_0 t}}{2} \left[\left(1 + \left| \frac{\bar{A}_f}{A_f} \right| \left| \frac{q}{p} \right|^2 \right) + \left(1 - \left| \frac{\bar{A}_f}{A_f} \right| \left| \frac{q}{p} \right|^2 \right) - \right. \\ \left. - 2 \eta_f^{CP} \left| \frac{\bar{A}_f}{A_f} \right| \left| \frac{q}{p} \right| \cos \varphi \cdot y \Gamma_0 t - 2 \eta_f^{CP} \left| \frac{\bar{A}_f}{A_f} \right| \left| \frac{q}{p} \right| \sin \varphi \cdot x \Gamma_0 t \right]$$

The other now

$$\underline{\hat{T}_{\bar{D} \rightarrow \bar{D}_f}(t)} = |A_f|^2 \frac{e^{-\Gamma_0 t}}{2} \left| \frac{p}{q} \right|^2 \left[\left(1 + \left| \frac{q}{p} \right| \left| \frac{\bar{A}_f}{A_f} \right|^2 \right) - \left(1 - \left| \frac{q}{p} \right| \left| \frac{\bar{A}_f}{A_f} \right|^2 \right) - \right. \\ \left. - 2 \eta_f^{CP} \left| \frac{q}{p} \right| \left| \frac{\bar{A}_f}{A_f} \right| \cos \varphi \cdot y \Gamma_0 t + 2 \eta_f^{CP} \left| \frac{q}{p} \right| \left| \frac{\bar{A}_f}{A_f} \right| \sin \varphi \cdot x \Gamma_0 t \right]$$

So let's conclude

$$\underline{\hat{T}_{D \rightarrow D_f}(t)} = |A_f|^2 e^{-\Gamma_0 t} \left[1 - \eta_f^{CP} \left| \frac{q}{p} \right| \left| \frac{\bar{A}_f}{A_f} \right| \cos \varphi \cdot y \Gamma_0 t - \eta_f^{CP} \left| \frac{q}{p} \right| \left| \frac{\bar{A}_f}{A_f} \right| \sin \varphi \cdot x \Gamma_0 t \right]$$

$$\underline{\hat{T}_{\bar{D} \rightarrow \bar{D}_f}(t)} = |A_f|^2 e^{-\Gamma_0 t} \left| \frac{p}{q} \right|^2 \left[\left| \frac{q}{p} \right|^2 \left| \frac{\bar{A}_f}{A_f} \right|^2 - \eta_f^{CP} \left| \frac{q}{p} \right| \left| \frac{\bar{A}_f}{A_f} \right| \cos \varphi \cdot y \Gamma_0 t \right. \\ \left. + \eta_f^{CP} \left| \frac{q}{p} \right| \left| \frac{\bar{A}_f}{A_f} \right| \sin \varphi \cdot x \Gamma_0 t \right]$$

we consider $\left| \frac{\bar{A}_f}{A_f} \right| \simeq 1$ (but it's not GREAT, we do that because now we know that CP violation exist)

Applying the approximation

$$\hat{T}_{D \rightarrow D_f}(t) = |A_f|^2 e^{-\Gamma_0 t} \left[1 - \eta_f^{CP} \Gamma_0 t \left(\overbrace{\left| \frac{q}{p} \right| y \cos \varphi + x \left| \frac{q}{p} \right| \sin \varphi}^{\alpha} \right) \right]$$

$$\hat{T}_{\bar{D} \rightarrow \bar{D}_f}(t) = |A_f|^2 e^{-\Gamma_0 t} \left[1 - \eta_f^{CP} \Gamma_0 t \cdot \overbrace{\left(\left| \frac{p}{q} \right| y \cos \varphi - \left| \frac{p}{q} \right| \sin \varphi \right)}^{\beta} \right]$$

Then we have

$$\Gamma_{D^0 \rightarrow f}(t) = |A_f|^2 e^{-\Gamma_0 t} [1 + \alpha \Gamma_0 t]$$

$$\Gamma_{\bar{D}^0 \rightarrow f}(t) = |A_f|^2 e^{-\Gamma_0 t} [1 + \beta \Gamma_0 t]$$

Knowing that $\eta_f^{CP} = -1$ and $e^{\alpha x} \simeq 1 + \alpha x$

$$\Gamma_{D^0 \rightarrow f}(t) = |A_f|^2 e^{-\Gamma_0 t} e^{\alpha \Gamma_0 t}$$

$$\Gamma_{\bar{D}^0 \rightarrow f}(t) = |A_f|^2 e^{-\Gamma_0 t} e^{\beta \Gamma_0 t}$$

rewriting them

$$\Gamma_{D^0 \rightarrow f} = |A_f|^2 e^{-\Gamma_0(1-\alpha)t}$$

$$\Gamma_{\bar{D}^0 \rightarrow f} = |A_f|^2 e^{-\Gamma_0(1-\beta)t}$$

$$\left. \begin{array}{l} \Gamma_D(1-\alpha) = \hat{\Gamma}_{D^0} \\ \Gamma_D(1-\beta) = \hat{\Gamma}_{\bar{D}^0} \end{array} \right\} \quad \begin{array}{l} \Gamma_{D^0 \rightarrow f}(t) = |A_f|^2 e^{-\hat{\Gamma}_{D^0} t} \\ \Gamma_{\bar{D}^0 \rightarrow f}(t) = |A_f|^2 e^{-\hat{\Gamma}_{\bar{D}^0} t} \end{array}$$

$$\eta_f^{CP} = -1$$

$$D^0 \rightarrow K^- K^+$$

$$D^0 \rightarrow \pi^- \pi^+$$

So now we can write the observable

$$\begin{aligned} \Delta\chi &= \frac{\bar{\Gamma}_{D^0} - \hat{\Gamma}_{D^0}}{2\hat{\Gamma}_{D^0}} = \frac{\cancel{\Gamma_D}(1-\beta) - \cancel{\Gamma_D}(1-\alpha)}{2\cancel{\Gamma_D}} \\ &= \frac{\alpha - \beta}{2} \end{aligned}$$

Remembering α, β

$$\alpha = \left| \frac{q}{p} \right| y \cos \varphi + \left| \frac{q}{p} \right| x \sin \varphi$$

$$\beta = \left| \frac{p}{q} \right| y \cos \varphi - \left| \frac{p}{q} \right| x \sin \varphi$$

$$\leadsto \frac{\alpha - \beta}{2} = \frac{x}{2} \left| \frac{q}{p} \right| \sin \varphi + \frac{y}{2} \left| \frac{q}{p} \right| \cos \varphi + \frac{x}{2} \left| \frac{p}{q} \right| \sin \varphi - \frac{y}{2} \left| \frac{p}{q} \right| \cos \varphi$$

$$\Delta y = \frac{x}{2} \left(\left| \frac{q}{p} \right| + \left| \frac{p}{q} \right| \right) \sin \varphi + \frac{y}{2} \left(\left| \frac{q}{p} \right| - \left| \frac{p}{q} \right| \right) \cos \varphi$$

There is another observable

$$y_{\text{cf}} = \frac{\hat{\Gamma}_{00}^{\circ} + \hat{\Gamma}_{00}}{2 \Gamma_0} = \frac{[(1-\beta) + (1-\alpha)] \Gamma_0}{2 \Gamma_0} - 1$$

$$= \frac{2 - \alpha - \beta}{2} - 1 = 1 - \frac{\alpha}{2} - \frac{\beta}{2} - 1 = -\frac{\alpha + \beta}{2}$$